This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 1126

Unique Paper Code : 235203 G

Name of the Paper : Analysis-II [MAHT 202]

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : II

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *three* parts of each question.

All questions are compulsory.

1. (*a*) Use the $\in - \delta$ definition of the limit to show that

$$\lim_{x \to c} x^2 = c^2, c \in R.$$

(*b*) Let $A \subseteq R$, let $f : A \to R$ and let c be a cluster point of A. If

$$\lim_{x \to c} f(x) < 0$$

then prove that there exists a neighbourhood $V_\delta(c)$ of c such that $f(x) < 0$ for all $x \in A \cap V_\delta(c)$, $x \neq c$.

P.T.O.

(*c*) Let f and g be continuous from R to R and suppose that $f(r) \geq g(r)$ for every rational numbers then prove that $f(x) \geq g(x)$ for all $x \in$ R.

(*d*) State Bolzano's Intermediate value theorem and hence prove that $xe^x = 2$ for some x in $[0, 1]$. 5,5,5,5

3\. (*a*) Let $I = [a, b]$ be an interval and let $f : I \to$ R be continuous on I. Then prove that f is uniformly continuous on I.

(*b*) Show that the function $f(x) = 1/x$ is uniformly continuous on $[a, \infty[$, $a > 0$ but is not uniformly continuous on $]0, \infty[$.

(*c*) Given that the function $f(x) = x^3 + 2x + 1$ for $x \in$ R has an inverse f^{-1} on R, find the value of $(f^{-1})'(y)$ at the points corresponding to $x = 0, 1$.

(*d*) Prove that if $f : I \to$ R has a derivative at $c \in$ I, then f is continuous at c. Is the converse true ? Justify your answer. 5,5,5,5

P.T.O.

4. (*a*) Let c be an interior point of the in[terval] I at which $f : I \to \mathbb{R}$ has a relative extremum. If [the derivative of f] at c exists then prove that :

$$f'(c) = 0.$$

Can f has a relative extremum at 'c' [if f is not] differentiable at 'c' ? Justify.

(*b*) Using the mean value theorem, prove that :

$$\frac{x-1}{x} < \log(x) < x - 1 \text{ for } x > 1.$$

(*c*) Find the point of relative extrema of the fun[ction]

$$f(x) = x(x - 8)^{1/3} \text{ for } 0 < x < 9.$$

(*d*) State Darboux theorem. Let $f : [0, 2] \to \mathbb{R}$ is [continuous] on [0, 2] and differentiable on]0, 2[and that $f(1) = 2$, $f(2) = 2$ then show that there exists c [...] such that :

$$f'(c) = 3/2.$$

5. (*a*) Obtain Maclaurin's series expansion for the function $\sin(4x)$.

(*b*) Using Taylor's theorem prove that :

$$1+\frac{x}{2}-\frac{x^2}{8}<\sqrt{1+x}<1+\frac{x}{2} \text{ for } x>0.$$

(*c*) Define Radius of convergence of the power series :

$$\sum_{n=0}^{\infty} a_n x^n.$$

Find the Radius of convergence and the exact interval of convergence for the power series :

$$\sum_{n=1}^{\infty} \frac{x^n}{n^2}.$$

(*d*) Check which of the following functions are convex :

(*i*) $|x|, x \in [-2, 5]$

(*ii*) $ax^3 + 2x + 3, a < 0, x \in [-1, 1]$. 5,5,5,5 (*c*) Using Sequantial Criterion for limi

$$\lim_{x \to 0} \sin(1/x^2)$$

does not exist in R.

(*d*) Using definition, prove that :

(*i*) $\lim_{x \to 0} \left(\frac{1}{x^2}\right) = \infty$

(*ii*) $\lim_{x \to \infty} 1/x = 0.$

2. (*a*) Prove that a function $f : A \to R$ is c

$c \in A$ if and only if for every seq

converges to c, the sequence $(f(x_n))$

(*b*) Let $A \subseteq R$, let $f : A \to R$ be co

$c \in R$. Show that for any $\in > 0$ there ex

$V_\delta(c)$ of c such that if $x, y \in A \cap$

$$|f(x) - f(y)| < \in.$$

[This question paper contains 7 printed pages]

Your Roll No.	: ....
Sl. No. of Q. Paper	: **1823** **GC-4**
Unique Paper Code	: 32351201
Name of the Course	: **B.Sc.(Hons.) Mathematics**
Name of the Paper	: Real Analysis
Semester	: II

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(c) **All** questions are compulsory.

(b) Attempt any **two** parts from each question.

1. (a) Define Infimum and Supremum of a non-empty subset of R.

Find infimum and supremum of the set

$$S = \left\{1 - \frac{(-1)^n}{n} : n \in N\right\}.$$ 5

P.T.O.

(b) Prove that a number u is the supremum of a non-empty subset S or R if and only if :

(i) $S \leq u \quad \forall s \in S$.

(ii) For any $\in > 0$, there exists $s_{\in} \in S$ such that $u - \in < S_{\in}$. 5

(c) State Archimedean Property of Real numbers. Prove that if $S = \left\{\frac{1}{n} : n \in N\right\}$, then inf S=0. 5

2. (a) Let A and B be bounded non-empty subsets of R. Define :

$A + B = \{a + b : a \in A \text{ and } b \in B\}$

Prove that inf (A+B) = inf A+inf B. 5

(b) State Density Theorem. Show that if x and y are real numbers with x < y, then there exists an irrational number z such that

$x < z < y$. 5

(c) Define limit point of a set. Find limit points of] 0, 1[. 5

3. (a) Define the convergence of a sequence (x_n) of real numbers. Show that if (x_n) is a convergent sequence of real numbers such that $x_n \geq 0 \; \forall n \in N$, then $x = \lim x_n \geq 0$

(b) Using the definition of the limit of a sequence, find the following limits :

(i) $\lim_{n \to \infty} \left(\frac{3n+1}{2n+5} \right)$.

(ii) $\lim_{n \to \infty} \left(\frac{\sqrt{n}}{n+1} \right)$. 5

(c) Prove that $\lim_{n \to \infty} n^{1/n} = 1$.

5

4. (a) Let (x_n) be a sequence of real numbers that converges to x and suppose that $x_n \geq 0 \forall n \in N$. Show that the sequence $\sqrt{x_n}$ converges to $\sqrt{x}$. 5

(b) Prove that every monotonically increasing bounded above sequence is convergent. 5

(c) If $x_1 < x_2$ are arbitrary real numbers and $x_n = \frac{1}{2}(x_{n-2} + x_{n-1})$ for n>2, show that (x_n) is convergent. What is its limit ? 5

5. (a) Define a Cauchy Sequence. Is the sequence (x_n) a Cauchy Sequence, where $x_n = 1 + \frac{1}{2!} + \ldots\ldots + \frac{1}{n!}$? Justify your answer. $7\frac{1}{2}$

(b) State and prove Bolzano Weierstrass Theorem for sequences. Justify the theorem with an example. $7\frac{1}{2}$

(c) (i) Show that if (x_n) is unbounded, then there exists a subsequence (x_{nk}) such that : $\lim\left(\frac{1}{x_{nk}}\right)=0$. 5

(ii) Show that the sequence $\left(1, \frac{1}{2}, 3, \frac{1}{4}, \ldots\ldots\right)$ is divergent.

$2\frac{1}{2}$

6. (a) If $\sum_{n=1}^{\infty} x_n$ converges, then prove that $\lim_{n\to\infty} x_n = 0$. Does the converse hold ?

5

(b) Test the convergence of any **two** of the following series : 5

(i) $\sum \frac{n+1}{n2^n}$

(ii) $\sum \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

P.T.O.

(iii) $\sum \frac{n^2}{n!}$

(c) State the Alternating Series Test. Show that the alternating series $\sum \frac{(-1)^n}{n}$ is convergent. 5

7. (a) Let $0 \leq a_n \leq b_n \forall n$. Show that : 5

(i) If $\sum_{n=1}^{\infty} b_n$ converges, then so does $\sum_{n=1}^{\infty} a_n$.

(ii) If $\sum_{n=1}^{\infty} a_n$ diverges, then so does $\sum_{n=1}^{\infty} b_n$.

(b) Show that every absolutely convergent series is convergent but the converse is not true. 5

(c) State Integral Test. Find the condition of convergence of the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n^p}$. 5

[This question paper contains 8 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **1824** **GC-4**

Unique Paper Code : 32351202

Name of the Course : **B.Sc.(Hons.) Mathematics–I**

Name of the Paper : Differential Equations.

Semester : II

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Use of non-programmable scientific calculator is allowed.

SECTION – A

1. Attempt any **three** parts, each part is of **5** marks.

(a) Solve the initial value problem :

$$x\frac{dy}{dx}+y = xy^{3/2},\ y(1)=4.$$

(b) Determine the most general function M (x, y) such that the equation

M (x, y) dx + (x^2 y^3 + x^4y) dy =0, is exact and hence solve it.

P.T.O.

(c) Solve the differential equation :

$(x^2 - 3y^2)\,dx + 2xy\,dy = 0.$

(d) Check the exactness of the differential equation :

$(3y + 4xy^2)\,dx + (2x + 3x^2y)\,dy = 0.$

Hence solve it by finding the integrating factor of the form $x^p y^q$.

2. Attempt any **two** parts; each part is of **5** marks.

(a) A certain moon rock was found to contain equal numbers of potassium and argon atoms. Assume that all the argon is the result of radioactive decay of potassium (its half-life is about 1.28×10^9 years) that one of every nine potassium atom disintegrations yields an argon atom. What is the age of the rock, measured from the time it contained only potassium ?

(b) A hemispherical bowl has top radius 4 ft and at time t = 0 is full of water. At that moment a circular hole with diameter 1 inch is opened in the bottom of the tank. How long will it take for all the water to drain from the tank ?

(c) A motor boat starts from rest (initial velocity $v(0) = v_0 = 0$). Its motor provides a constant acceleration of 4 ft/s^2, but water resistance causes a deceleration of $\frac{v^2}{400}$ ft/s^2. Find v when t = 10 s, and also find the limiting velocity as $t \to +\infty$ (that is, the maximum possible speed of the boat).

SECTION – B

3. Attempt any **two** parts; each part is of **7.5** marks.

(a) Consider the American system of two lakes: Lake Erie feeding into Lake Ontario. Assuming that volume in each lake to remain constant and that Lake Erie is the only source of pollution for Lake Ontario.

(i) Write down a differential equation describing the concentration of pollution in each of two lakes, using the variables V for volume, F for flow, c(t) for concentration at time t and subscripts 1 for Lake Erie and 2 for Lake Ontario.

P.T.O.

(ii) Suppose that only unpolluted water flows into Lake Erie. How does this change the model proposed ?

(iii) Solve the system of equations to get expression for the pollution concentration $c_1(t)$ and $c_2(t)$.

(b) The following model describes the levels of a drug in a patient taking a course of cold pills :

$$\frac{dx}{dt} = I - k_1 x, \quad x(0) = 0$$

$$\frac{dy}{dt} = k_1 x - k_2 y, \quad y(0) = 0$$

Where k_1 and k_2 ($k_1 > 0$, $k_1 > 0$ and $k_1 \neq k_2$) describes rate at which the drug moves between the two sequential compartments (the GI-tract and the bloodstream) and I denotes the amount of drug released into the GI-tract in each step. At time t, x and y are the levels of the drug in the GI-tract and bloodstream respectively.

(i) Find solution expressions for x and y which satisfies this pair of differential equations.

(ii) Find the levels of the drug in the GI-tract and the bloodstream as $t \to \infty$.

(c) In view of the potentially disastrous effects of overfishing causing a population to become extinct, some governments impose quotas which vary depending on estimates of the population at the current time. One harvesting model that takes this into account is

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) - h_0 X.$$

(i) Find the non-zero equilibrium population.

(ii) At what critical harvesting rate can extinction occur ?

SECTION – C

4. Attempt any **four** parts; each part is of **5** marks.

(a) Use the method of variation of parameters to find a particular solution of the differential equation

$$y'' - 4y' + 4y = 2e^x.$$

P.T.O.

(b) Use the method of undetermined coefficients to solve the differential equation

$$y'' + y = \sin x.$$

(c) A body with mass $m = \frac{1}{2}$ kg is attached to the end of the spring that is stretched 2 m (meters) by a force of 100 N (Newtons). It is set in motion with initial position $x_0 = 1$ m and initial velocity $v_0 = -5$m/s. Find the position function of the body as well as the amplitude, frequency and period of the oscillation.

(d) Show that the two solutions $y_1(x) = e^x \cos x$ and $y_2(x) = e^x \sin x$ of the differential equation $y'' - 2y' + 2y = 0$ are linearly independent on the open interval I. Then find a particular solution of the above differential equation with initial condition

$$y(0) = 1 \text{ and } y'(0) = 5.$$

(e) Find the general solution of the Euler equation $x^2 y'' + 7xy' + 25y = 0$.

SECTION – D

5. Attempt any **two** parts; each part is of **7.5** marks.

(a) Consider a disease where all those who are infected remain contagious for life. Assume that there are no births and deaths :

(i) Write down suitable word equations for the rate of change of numbers of susceptibles and infectives. Hence develop a pair of differential equations.

(ii) Draw a sketch of typical phase-plane trajectories for this model. Determine the direction of travel along the trajectories.

(b) A simple model for a battle between two army red and blue, where both the army used aimed fire, is given by the coupled differential equations –

$$\frac{dR}{dt} = -a_1B, \frac{dB}{dt} = -a_2R$$

Where R and B are the number of soldiers in the red and blue army respectively and a_1 and a_2 are the positive constants.

P.T.O.

(i) Use the chain rule to find a relationship between R and B, given the initial numbers of soldiers for the two armies are r_0 and b_0 respectively.

(ii) Draw a rough sketch of phase-plane trajectories.

(iii) If both the army have equal attrition coefficients i.e. $a_1 = a_2$ and there are 10,000 soldiers in the red army and 8000 in blue army. Determine who wins if there is one battle between the two army.

(c) Consider the Lotka – Volterra model describing the simple predator prey model :

$$\frac{dx}{dt} = b_1X - c_1XY \quad \text{and} \quad \frac{dY}{dt} = c_2XY - a_2Y$$

where b_1, c_1, c_2, a_2 are positive constants and X and Y denotes the prey and predator populations respectively at time t.

(i) Find the equilibrium solutions of the above model.

(ii) Find the directions of trajectories in the phase plane.

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2447**

Unique Paper Code : **32355202** **GC-4**

Name of the Paper : **Linear Algebra**

Name of the Course : **Generic Elective : Mathematics for Honours**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

1. (*a*) If x and y are vectors in $\mathbf{R}^n$, then prove that :

(*i*) $\|x + y\|^2 = \|x\|^2 + \|y\|^2$ if and only if $x \cdot y = 0$, and

(*ii*) $x \cdot y = \frac{1}{4}\left(\|x + y\|^2 - \|x - y\|^2\right)$. 3+3

(*b*) Let x and y be non-zero vectors in $\mathbf{R}^n$, then prove that :

$$\|x + y\| = \|x\| + \|y\|$$

if and only if $y = cx$, for some $c > 0$. 6

P.T.O.

(c) Using Gauss-Jordan method, find the complete solution set for the following system of homogeneous linear equations : 6

$$4x_1 - 8x_2 - 2x_3 = 0$$

$$3x_1 - 5x_2 - 2x_3 = 0$$

$$2x_1 - 8x_2 + x_3 = 0.$$

2. (a) Find the reduced row echelon form matrix B of the following matrix : 4+2½

$$A = \begin{pmatrix} 4 & 0 & -20 \\ -2 & 0 & 11 \\ 3 & 1 & -15 \end{pmatrix}$$

and then give a sequence of row operations that converts B back to A.

(b) Find the characteristic polynomial and eigenvalues of the matrix : 4+2½

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 2 & -4 \\ 3 & -4 & 7 \end{pmatrix}.$$

Is A diagonalizable ? Justify.

(c) Let V be the set $\mathbf{R}^2$ with operations addition and scalar multiplication for x, y, w, z and a in $\mathbf{R}$ defined by :

$$[x, y] \oplus [w, z] = [x + w - 2, y + z + 3], \text{ and}$$

$$a \odot [x, y] = [ax - 2a + 2, ay + 3a - 3].$$

Prove that V is a vector space over $\mathbf{R}$. Find the zero vector in V and the additive inverse of each vector in V. 4+2½

3. (a) Prove that the set S = {[3, 1, −1], [5, 2, −2], [2, 2, −1]} is linearly independent in $\mathbf{R}^3$. Examine whether S forms a basis for $\mathbf{R}^3$? 4+2

(b) Find a basis and the dimension for the subspace W of $\mathbf{R}^3$ defined by : 6

$$W = \{[x, y, z] \in \mathbf{R}^3 : 2x - 3y + z = 0\}.$$

(c) Let S = {[1, 2], [0, 1]} and T = {[1, 1], [2, 3]} be two ordered bases for $\mathbf{R}^2$. Let $v = [1, 5]$. Find the coordinate vector $[v]_S$ and hence find $[v]_T$ using the transition matrix $Q_{T \leftarrow S}$ from S-basis to T-basis. 3+3

4. (*a*) Using rank, find whether the non-homogeneous linear system $Ax = b$, where :

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & -3 & 4 \\ 2 & -1 & 7 \end{pmatrix}, b = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}.$$

has a solution or not. 6

(*b*) Suppose $L : \mathbf{R}^3 \to \mathbf{R}^2$ is a linear transformation with $L([1, -1, 0]) = [2, 1]$, $L([0, 1, -1]) = [-1, 3]$ and $L([0, 1, 0]) = [0, 1]$. Find $L([-1, 1, 2])$. Also give formula for $L([x, y, z])$, for any $[x, y, z] \in \mathbf{R}^3$. 4+2

(*c*) Let $L : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear operator given by $L([x, y]) = [2x - y, x - 3y]$. Find the matrix for L with respect to the basis $\{[4, -1], [-7, 2]\}$ using the method of similarity. 6

5. (*a*) Consider the linear transformation $L : P_2 \to \mathbf{R}$ defined by :

$$L(p(x)) = \int_0^1 p(x)\, dx,$$

where P_2 is the vector space of polynomials of degree 2 or less. Show that L is onto but not one-to-one.

(*b*) Let W be the subspace of $\mathbf{R}^3$ whose vectors lie in the plane $2x + y + z = 0$. Find the minimum distance from the point P(–6, 10, 5) to W. 6

(*c*) Find a least squares solution for the linear system $Ax = b$, where : 6

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \\ 4 & 1 \end{pmatrix}, \; b = \begin{pmatrix} 0 \\ 5 \\ 4 \end{pmatrix}.$$

(*a*) Use the similarity method to show that a rotation about the point (1, –1) through an angle $\theta = 90°$, followed by a reflection about the line $x = 1$ is represented by the matrix $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix}$. 6½

(*b*) Let $L : P_2 \to \mathbf{R}^2$ be the linear transformation given by :

$$L(p(x)) = [p(1), p'(1)],$$

where P_2 is the vector space of polynomials of degree 2 or less. Find a basis for ker(L) and a basis for range(L), and also verify the dimension theorem. 4+2½

(c) For the subspace $W = \{[x, y, z] \in \mathbf{R}^3 : 3x - y + 4z = 0\}$ of $\mathbf{R}^3$, find the orthogonal complement $W^{\perp}$ and veirfy that $\dim(W) + \dim(W^{\perp}) = \dim(\mathbf{R}^3)$. 4+2½

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1567 F-8

Unique Paper Code : 2351401

Name of the Paper : ANALYSIS III (Riemann Integration and Series of Functions)

Name of the Course : B.Sc. (H.) Mathematics

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two parts from each question.
3. All questions are compulsory.

1. (a) Let f be a bounded function on [a, b]. Show that $L(f) \leq U(f)$, where L(f) is the lower Darboux integral and U(f) is the upper Darboux integral on [a,b]. (6)

(b) Let f be a continuous function on R and define

$$F(x) = \int_{x-1}^{x+1} f(t)\,dt$$ for $x \in R$. Show that F is differentiable

function on R. Compute F′. (6)

(i) $\int_1^\infty \frac{1}{x\sqrt{x+1}}\,dx$

(ii) $\int_1^\infty e^{t^2}\,dt$

(3+3)

4. (a) Let $\langle f_n \rangle$ be a sequence of continuous functions on $A \subseteq R$ and suppose that (f_n) converges uniformly on A to $f: A \to R$. Then show that f is continuous on A.

(6½)

(b) Let $f_n(x) = \frac{nx}{1+n^2x^2}$ for $x \geq 0$. Show that the sequence $\langle f_n \rangle$ converges only pointwise on $[0, \infty[$ and converges uniformly on $[a, \infty[$, $a > 0$.

(6½)

(c) Let $f_n(x) = \frac{x}{1+nx^2}$ for $x \in R$, $n \in N$. Show that the sequence $\langle f_n \rangle$ converges uniformly on R.

(6½)

5. (a) State and prove Weierstrass-M test for the uniform convergence of a series of functions.

(2,5)

(b) Show that the series of function $\sum_{n=1}^{\infty} \sin\left(\frac{x}{n^2}\right)$ converges uniformly for $|x| \leq a$, $a > 0$ and is not uniformly convergent on R.

(7)

(c) Suppose f and g are continuous function[s] that $g(x) \geq 0$ for all $x \in [a,b]$. Prove th[at] c in [a, b] such that :

$$\int_a^b f(t)g(t)\,dt = f(c)\int_a^b g(t)\,dt.$$

2. (a) Prove that every monotonic function integrable.

(b) Let f be defined on [0, b] as $f(x) = \begin{cases} x, \\ 0, \end{cases}$

Calculate upper and lower Darboux inte[grals] [0, b]. Is f integrable on [0, b] ?

(c) Let f be a bounded function on [a,b] If partitions of [a,b] such that $P \subseteq Q$, where exactly one point extra than the points of that $L(f, P) \leq L(f, Q) \leq U(f, Q) \leq U(f, P)$

3. (a) Show that the improper integral $\int_1^{\infty} \frac{\sin t}{t}\,dt$ but not absolutely convergent.

(b) Show that the improper integral $\int_0^1 x^{m-}$ converges if and only if $m > 0$, $n > 0$.

(c) Examine the convergence of the followin[g] integrals :

(c) Show that the sequence $< x^2e^{-ax} >$ c
on $[0, \infty[$.

6. (a) Let $S(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ and $C(x)$
$x \in R$. Prove that :

(i) $S' = C$ and $C' = -S$

(ii) $(S^2 + C^2)' = 0$

(iii) $S^2 + C^2 = 1$.

(b) Show that the function $E(x) = \sum_{n=}^{\infty}$
defined and is differentiable on R w
$E'(x) = E(x)$ for all $x \in R$ and $E(x +$
all $x, y \in R$.

(c) Find the radius of convergence of th
series :

(i) $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^n$

(ii) $\sum_{n=1}^{\infty} n^{-\sqrt{n}} x^n$

(3) copies

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1566 F-8

Unique Paper Code : 2351402

Name of the Paper : Algebra – III (Ring Theory and Linear Algebra – I)

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two parts from each question.

1. (a) Let $M_2(\mathbb{Z})$ be the ring of all 2×2 matrices over $\mathbb{Z}$, the set of all integers and

$$R = \left\{ \begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix} : a, b \in \mathbb{Z} \right\}.$$

Prove or disprove that R is a subring of $M_2(\mathbb{Z})$.

(b) Define an idempotent element of a ring. Show that in a commutative ring of characteristic 2, the idempotents form a subring.

P.T.O.

(c) Let R be a ring with unity 1. If 1 has i under addition, then prove that the characte 0. If 1 has order n under addition, then p characteristic of R is n.

2. (a) Show that $\mathbb{R}[x]/\langle x^2 + 1\rangle$ is a field.

(b) Define Prime ideal and Maximal ideal of that every maximal ideal is prime in a com with unity. Is the converse true? Justify.

(c) Determine all the distinct elements of th R/I, where R = $M_2(\mathbb{Z})$, the ring of all 2 over $\mathbb{Z}$ and I is the ideal of R consisting o with even entries.

3. (a) Determine all ring homomorphisms from

(b) State and prove the First Isomorphism The

(c) Show that a homomorphism from a field o more than one element must be an isom

4. (a) Prove that the Span of any subset S of V is a subspace of V. Moreover, any that contains S must also contain the Sp

(b) Let V be a vector space over a field of characteristic not equal to 2. Let u, v and w be distinct vectors in V. Prove that {u, v, w} is linearly independent if and only {u + v, u + w, v + w} is linearly independent.

(c) The vectors {(2, –3, 1), (1, 4, –2), (–8, 12, –4), (1, 37, –17), (–3, –5, 8)} generate $\mathbb{R}^3$. Find a subset of this set that is a basis for $\mathbb{R}^3$. (6.5,6.5,6.5)

5. (a) Let V and W be finite dimensional vector spaces over a field F and $T : V \to W$ be a linear transformation. Prove that nullity (T) + rank (T) = dim V.

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(a_1, a_2) = (a_1 - a_2, a_1, 2a_1 + a_2)$. Let β be the standard ordered basis for $\mathbb{R}^2$ and γ = {(1,1,0), (0,1,1), (2,2,3)}. Compute $[T]_\beta^\gamma$.

(c) Let V be a vector space and let $T : V \to V$ be linear. Prove that $T^2 = T_0$ if and only if $R(T) \subseteq N(T)$.

(6.5,6.5,6.5)

6. (a) Let V and W be finite dimensional vector spaces over the same field F. Prove that V is isomorphic to W if and only if dim V = dim W.

(b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x_1, x_2, x_3) = (3x_1 - 2x_3, x_2, 3x_1 + 4x_2)$. Prove that T is invertible and find $T^{-1}(x_1, x_2, x_3)$.

(c) Let T be the linear operator

$$T\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2a+b \\ a-3b \end{bmatrix}$$. Let β be the sta

for $\mathbb{R}^2$ and let $\beta' = \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}\right\}$. Find

Q such that $[T]_{\beta'} = Q^{-1}[T]_{\beta}Q$.

(This question paper contains 3 printed pages)

Sl. No. of Q.P.: 1568

Roll No.....................

Unique Paper Code : 2351403

Name of the Paper : Differential Equations - II (PDE & System of ODE)

E-8

Name of the Course : B.Sc. (II) Mathematics

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the six questions are compulsory.

Attempt any two parts from each question.

1. (a) Find the partial differential equation arising out of the family of right circular cones whose axes coincide with the z-axis: $x^2 + y^2 = (z - c)^2 \tan^2 \alpha$.

(b) Define linear first order, quasi-linear partial differential equations with example. Give the geometrical interpretation of the same.

(c) Solve the initial-value problem: $3u_x + 2u_y = 0$, with $u(x, 0) = \sin x$, (12)

2. (a) Apply the method of separation of variables $u(x, y) = f(x)\, g(y)$ to solve the equation $u_x + u = u_y$, $u(x, 0) = 4e^{-3x}$.

(b) Derive two - dimensional wave equation.

(c) Transform the equation $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = 0$ into the canonical form and then find the general solution.

(12)

3. (a) Obtain the solution of the Cauchy problem for the non-homogeneous wave equation:

$$u_{tt} = c^2 u_{xx} + h^*(x, t)$$

with the initial conditions:

$$u(x,0) = f(x),$$
$$u_t(x,0) = g'(x).$$

(b) Solve the initial-value problem:

$$u_{xx} + 2u_{xy} - 3u_{yy} = 0,$$
$$u(x,0) = \sin x,$$
$$u_y(x,0) = x.$$

(c) Determine the solution of the initial boundary-value problem:

$$u_{tt} = 4u_{xx}, 0 < x < \infty, t > 0,$$
$$u(x,0) = x^4, 0 \le x < \infty,$$
$$u_t(x,0) = 0, 0 \le x < \infty$$
$$u(0,t) = 0, t \ge 0.$$

(12)

4(a) Solve the initial boundary-value problem by the method of separation of variables:

$$u_{tt} = c^2 u_{xx}, 0 < x < 1, t > 0,$$
$$u(x,0) = x(1-x), u_t(x,0) = 0; 0 \le x \le 1,$$
$$u(0,t) = u(1,t) = 0, t > 0.$$

(b) Obtain the solution of the initial boundary-value problem:

$$u_t = ku_{xx}, 0 < x < \pi, t > 0,$$
$$u(x,0) = \sin^2 x, 0 \le x \le \pi,$$
$$u(0,t) = u(\pi,t) = 0, 0 \le t < \infty.$$

(c) Solve the Cauchy problem for the system:

$$u_{tt} - 9u_{xx} = 0, x \in \mathbb{R}, t > 0,$$
$$u(x,0) = \cos x, u_t(x,0) = \sin 2x \text{ for } x \in \mathbb{R}.$$

(13)

Q 5.(a) Use the operator method to find the general solution of the linear system of equations:

$$\frac{d^2x}{dt^2} + \frac{dy}{dt} = t + 1, \quad \frac{dx}{dt} + \frac{dy}{dt} - 3x + y = 2t - 1.$$

(b) Find the particular solution of the linear system that satisfies the given initial conditions:

$$\frac{dx}{dt} = 6x - 4y, \; x(0) = 2, \quad \frac{dy}{dt} = x + 2y, \; y(0) = 3.$$

(c) Prove that the ordered pair of functions defined for all t by $(2e^{2t}, -3e^{2t})$ is a solution of the Homogeneous linear system of equations:

$$\frac{dx}{dt} = 5x + 2y, \quad \frac{dy}{dt} = 3x + 4y.$$

(13)

Q6.(a) Using the method of successive approximations, find the sequence of functions that approaches the exact solution of the initial-value problem:

$$\frac{dy}{dx} = x^2 + y^2, \; y(0) = 1.$$

(b) Apply the modified Euler method to the initial-value problem:

$$\frac{dy}{dx} = x + y, \; y(0) = 0.$$

Apply the method to approximate the values of the solution y at x=0, and x=1.0 using h = 0.5.

(c) Apply the Runge-Kutta method to the initial-value problem:

$$\frac{dy}{dx} = 2x - y, \; y(0) = 3.$$

Apply the method to approximate the values of the solution y at x=0.4, using h = 0.2.

Carry the intermediate calculations in each step to five figures after the decimal points. **(13)**

(This Question Paper contains 2 printed pages)

Roll No.:

100

Sr. No. of Question Paper: 1624

Unique Paper Code: 2362401

Name of the Course: Mathematics (~~Hons. II~~) UP Allied Course

F-8

Name of the Paper: Inventory & Production Management

Semester: ~~III~~ IV

Duration: 3 hours.

Max marks: 75

Instruction for the candidates

1. Write your roll number on the top immediately on receipt of this question paper.
2. Answer five questions in all.
3. All questions carry equal marks.
4. Simple calculators are allowed.

Q1(a) What do you mean by term 'Inventory'? State any five reasons for carrying inventory in an organization. (7)

(b) Obtain an expression for the EOQ for the deterministic demand, supply is instantaneous, lead time is zero, backorders are allowed and are fully backlogged. (8)

Q2(a) Demand for an item is constant at 150 units per month. The item can be made at a constant rate of 3500 units a year. Unit cost is Rs. 100, batch setup cost is Rs. 650 and holding cost is 20% of unit cost per year. What is the optimal batch size for the item? If production setup time is 14 days, when should this is started? (7)

(b) Find the optimal order quantity for which the price breaks are as follows:

$0 \le q < 500$ $\quad Rs.100.00$

$500 \le q < 750$ $\quad Rs.98.25$

$750 \le q$ $\quad Rs.95.75$

The monthly demand for the product is 1500 units, the cost of storage per unit per year is 20% of the unit cost and the ordering cost is Rs. 150 per order. (8)

Q3 (a) Derive the mathematical model for "All unit quantity discounts" when shortages are not allowed. (7)

(b) A store wants to improve the control of its stock and is looking at the possibility of using ABC analysis. Records from eight types of item show the current sales and costs as follows:

Item	No. of Sales	Cost (Rs.)
1	25	1400
2	150	14
3	30	680
4	80	20
5	10	1020
6	40	150
7	1000	20
8	100	30

Perform the ABC analysis. (8)

Q4. Derive the optimal inventory policy when there is constraint on:

(i) Average inventory

(ii) Floor Area (15)

Newspaper

Q5(a) Discuss the Newsboy problem. Formulate and derive the single period, discrete and stochastic demand model. (8)

(b) What is material requirement planning (MRP)? What are its advantages and disadvantages? (7)

Q6. Explain any three of the following:

1) Reorder level ~~and Reorder level~~
2) Various Methods of classifying Inventory
3) Difference between Deterministic and Stochastic Inventory Models
4) Various cost involved in Inventory Control

(3x5)

[This question paper contains 02 printed pages.]

Sr. No. of Question Paper : 1635 **Roll No.........**

Unique Paper Code : 2352301

Name of the Course : ~~Erstwhile FYUP (other than B.Sc. (H) Mathematics)~~ Allied Course Mathematics F-8

Name of the Paper : Calculus

Semester : IV

Duration: 3 Hours Maximum Marks:75

Instructions for Candidates

1) Write your Roll No. on the top immediately on receipt of this question paper.
2) Attempt any three parts of each section.

SECTION 1 $(6\frac{1}{2} \times 3)$

I. Given $f(x) = 2x - 2$, $x_o = -2$, $\varepsilon = 0.02$. Find $L = \lim_{x \to x_o} f(x)$. Then find a number $\delta > 0$ such that for all x, $0 < |x - x_o| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

II.

a. Find a linearization of $f(x) = \frac{1}{1-x}$ at 0.

b. The radius r of a circle increases from $a = 10m$ to $10.1m$. Use dA to estimate the increase in the circle's area A. Estimate the area of the enlarged circle and compare your estimate to the true area found by direct calculation.

III. Prove that $\lim_{x \to \infty} \frac{1}{x} = 0$.

IV. Evaluate $\lim_{x \to 0} \frac{\sqrt{x+1} - 1 - x/2}{x^2}$.

SECTION 2 $(6\frac{1}{2} \times 3)$

I. Sketch the graph of a function $f(x) = x^4 - 4x^3 + 10$.

II. A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of cylinder. The second plane crosses the first plane at $45°$ angle at the center of the cylinder. Find the volume of the wedge.

III. The region bounded by the curve $y = \sqrt{4x - x^2}$, the x-axis, and the line $x = 2$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

IV. Find the length of the curves in $y = x^{3/2}$ from $x = 0$ *to* $x = 1$.

SECTION 3

(6 X 3)

I. Sketch the graph of $r = cos2\theta$ in polar coordinates.

II. The vector $\boldsymbol{r}(t) = (3\cos t)\boldsymbol{i} + (3\sin t)\boldsymbol{j} + t^2\boldsymbol{k}$ gives the position of a moving body at time t. Find the body's speed and direction when $t = 2$. At what times, if any, the body's velocity and acceleration orthogonal?

III. Find the unit tangent vector of the helix

$$\boldsymbol{r}(t) = (\cos t \ + t\sin t)\boldsymbol{i} + (\sin t - t\cos t)\boldsymbol{j}, \qquad t > 0$$

IV. Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}.$$

SECTION 4

(6 X 3)

I. Use the Chain Rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t\,, y = \sin t$. What is the derivative's value at $t = \frac{\pi}{4}$? → $t = \frac{\pi}{4}$?

II. Find the derivative of $f(x, y) = x^2 + xy$ at $P_0(1,2)$ in the direction of unit vector

$$u = \left(\frac{1}{\sqrt{2}}\right)i + \left(\frac{1}{\sqrt{2}}\right)j.$$

III. Find the local extreme values of the function $f(x, y) = xy$.

IV. Let $r(t) = ti + t^2j + (t - 1)k$. Find $\lim_{t\to1}[r(t)\cdot(r'(t)\times r''(t))]$.

[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : **2814** **GC-4**

Unique Paper Code : 32351401

Name of the Paper : C-8 Partial Differential Equations

Name of the Course : **B.Sc. (Hons.) Mathematics – II**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory.
3. Marks of each part are indicated.

SECTION – I

1. Attempt any **two** parts out of the following :

(a) Find the solution of the Cauchy problem

$$u_x + xu_y = \left(y - \frac{1}{2}x^2\right)^2, \text{ with } u(0, y) = \exp(y). \quad (7\tfrac{1}{2})$$

(b) Using $v = \ln u$ and $v = f(x) + g(y)$, find the solution the Cauchy problem

$$y^2u_x^2 + x^2u_y^2 = (xyu)^2, \quad \text{with } u(x,0) = e^{x^2}.$$

(7

(c) Find the integral surfaces of the equation $uu_x + u_y$: for the following initial data : $x(s,0) = s$, $y(s,0) =$ $u(s,0) = s$.

(7

SECTION – II

2. Attempt any **one** part out of the following :

(a) State Conservation Law and derive the Burge Equation

$$u_t + uu_x = vu_{xx},$$

where v is the kinematic viscosity and $u(x,t)$ is the flu velocity field.

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(b) Derive the damped wave equation of a string

$$u_{tt} + au_t = c^2u_{xx},$$

where the damping force is proportional to the veloci and a is constant. Considering a restoring for proportional to the displacement of a string, show th the resulting equation is

(

$u_{tt} + au_t + bu = c^2u_{xx}$, where b is a constant.

Attempt any **two** parts out of the following :

(a) Find the characteristics, characteristic coordinates and reduce the equation given below to the canonical form :

$u_{xx} + x^2 u_{yy} = 0, \quad \text{for } x \neq 0.$ (6)

(b) Use the polar coordinates r and θ ($x = r\cos\theta$, $y = r\sin\theta$) to transform the Laplace equation $u_{xx} + u_{yy} = 0$ into the polar form

$$\nabla^2 u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0. \quad (6)$$

(c) Obtain the general solution of the equation given below :

$x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} + xyu_x + y^2 u_y = 0$ (6)

SECTION – III

Attempt any **three** parts out of the following :

(a) Determine the solution of the Cauchy problem given below :

$u_{tt} - 9u_{xx} = 0, \; x \in R, \; t > 0$

$u(x,0) = \cos x, \qquad u_t(x,0) = \sin 2x$ (7)

(b) Find the solution of the initial boundary- value problem

$u_{tt} = 4u_{xx}, \quad 0 < x < 1, \quad t > 0,$

$u(x,0) = 0, \quad 0 \le x \le 1,$

$u_t(x,0) = x(1-x), \quad 0 \le x \le 1,$

$u(0,t) = 0, \; u(1,t) = 0, \quad t \ge 0.$ (7)

(c) Solve the Cauchy problem for the non- homogeneous wave equation

$u_{tt} = c^2 u_{xx} + h^*(x,t),$

with the initial conditions

$u(x,0) = f(x), \quad u_t(x,0) = g^*(x)$ (7)

(d) Solve the characteristic initial- value problem

$xu_{xx} - x^3 u_{yy} - u_x = 0, \; x \neq 0,$

$u(x,y) = f(y) \text{ on } y - \frac{x^2}{2} = 0 \text{ for } 0 \le y \le 2,$

$u(x,y) = g(y) \text{ on } y + \frac{x^2}{2} = 4 \text{ for } 2 \le y \le 4,$ (7)

where $f(2) = g(2)$.

SECTION – IV

Attempt any three parts out of the following :

(a) Discuss the solution of the Heat Conduction Problem

$u_t = ku_{xx}, \ 0 < x < l, \ t > 0$

$u(0,t) = 0, \ t \geq 0$

$u(l,t) = 0, \ t \geq 0$

$u(x,0) = f(x), \ 0 \leq x \leq 1.$ (7)

(b) Solve using the method of separation of variables

$u_{tt} - c^2 u_{xx} = 0, \quad 0 < x < \pi, \quad t > 0,$

$u(x,0) = \sin x, \quad u_t(x,0) = x^2 - \pi x, \quad 0 \leq x \leq \pi,$

$u(0,t) = u(\pi,t) = 0, \quad t > 0.$ (7)

(c) Determine the solution of

$u_{tt} = c^2 u_{xx} + A \sinh x, \ 0 < x < l, \quad t > 0,$

$u_t(x,0) = 0, \quad u_t(x,0) = 0, \quad 0 \leq x \leq l,$

$u(0,t) = h, \quad u(l,t) = k, \quad t > 0$

where h, k and A are constants. (7)

(d) Find the temperature distribution in a rod of lengt
The faces are insulated, and the initial temperat
distribution is given by $x(l - x)$.

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2815**

Unique Paper Code : **32351402** GC-4

Name of the Paper : **C-9, Riemann Integration and Series of Functions**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

1. (*a*) Prove that a bounded function f on $[a, b]$ is integrable if and only if for each $\varepsilon > 0$, $\exists$ a $\delta > 0$ such that :

$$\text{mesh}(P) < \delta \Rightarrow U(f, P) - L(f, P) < \varepsilon,$$

$\forall$ partitions P of $[a, b]$. 6

(*b*) Show that every continuous function on $[a, b]$ is integrable. Is the converse true ? Justify. 6

(*c*) State and prove Intermediate Value Theorem for integrals. Show, with the help of an example, that the condition of continuity cannot be relaxed. 6

P.T.O.

2. (a) (i) Let f be a bounded function on $[a, b]$. Define the Darboux Integral $\int_a^b f$. 3

(ii) Give an example of a function f which is not integrable, for which $|f|$ is integrable. 3

(b) (i) State Fundamental Theorem of Calculus-II. 2

(ii) Let f be a continuous function on $\mathbf{R}$, and define :

$$F(x) = \int_{x-1}^{x+1} f(t)\, dt \text{ for } x \in \mathbf{R}$$

Show that F is differentiable on $\mathbf{R}$, and compute F'. 4

(c) (i) Define :

$$f(x) = [x] \text{ on } [0, 3].$$

Show that f is integrable, and evaluate $\int_0^3 f(x)\, dx$. 3

(ii) Define :

$$g : [0, 2] \rightarrow [0, 4] \text{ as } g(x) = x^2.$$

Let :

$$P = \{0, \frac{1}{2}, 1, \frac{3}{2}, 2\}.$$

Find $U(g, P)$. 3

3\. (a) Show that :

$$\int_0^\infty e^{-t} t^{x-1}\, dt$$

converges $\Leftrightarrow x > 0$. 7

(b) Determine the convergence or divergence of :

$$\int_0^\infty \frac{1}{e^x \sqrt{x}}\, dx.$$ 7

(c) In the following, determine the values of r for which the integral converges/diverges. Determine the values of integrals in case of convergence.

(i) $\int_1^\infty x^{-r}\, dx$

(ii) $\int_0^\infty x^{-r}\, dx$. 7

4\. (a) Let $\{f_n\}$ be a sequence of integrable functions on $[a, b]$ and suppose that $\{f_n\}$ converges uniformly on $[a, b]$ to f. Show that f is integrable. 6

(b) Let

$$f_n = \frac{1}{(1+x)^n}$$

for x in $[0, 1]$. Find the point-wise limit f of the sequence $\{f_n\}$ on $[0, 1]$.

Does $\{f_n\}$ converge uniformly to f on $[0, 1]$? Justify. 6

(c) Show that if $0 < b < 1$, then the convergence of the sequence :

$$f_n = \frac{x^n}{(1+x^n)},$$

for x in **R**, $x \geq 0$ is uniform on the interval $[0, b]$, but is not uniform on the interval $[0, 1]$. 6

5. (a) State and prove Weierstrass M-Test and hence examine the uniform convergence of the series :

$$\sum_{n=1}^{\infty} \frac{1}{(x^2+n^2)}, x \text{ in } \mathbf{R}.$$ 6

(b) Show that the series of functions :

$$\sum \frac{1}{(1+x^n)}, x \neq 0,$$

converges uniformly on $[a, \infty)$ for $a > 1$, but is not uniformly convergent on $(1, \infty)$. 6

(c) Let $f : A \to \mathbf{R}$, $A \subseteq \mathbf{R}$ be a bounded function on A. Define uniform norm $\| f \|$ on A. State criterion for uniform convergence of a sequence of bounded functions using uniform norm and hence examine for uniform convergence the sequence :

$$f_n = x^n;\ x \in [0, 1].$$ 6

(a) (i) Given :

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

for $|x| < 1$. Evaluate $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$. 2.5

(ii) Let

$$f(x) = \sum a_n x^n$$

for $|x| < R$. If $f(x) = f(-x), \forall |x| < R$, then show that $a_n = 0$ for all odd values of n. 4

(b) (i) If the power series $\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence R, then show that series :

$$\sum_{n=1}^{\infty} n a_n x^{n-1} \text{ and } \sum_{n=0}^{\infty} \frac{a_n x^{n+1}}{n+1}$$

also have the radius of convergence R. 4

(ii) Show that :

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \text{ for } x \in \mathbf{R}$$

is well-defined. Also show that :

$$f(x) = f'(x), \text{ for } x \in \mathbf{R}.$$ 2.5

(c) (i) Define interval of convergence for a power series. Find the radius of convergence and interval of convergence for the following power series :

$$\sum_{n=1}^{\infty} \frac{x^n}{n}, \quad \sum_{n=1}^{\infty} \frac{x^n}{n^2}.$$

(ii) State Weierstrass's Approximation Theorem.

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 2816

Unique Paper Code : 32351403 GC-4

Name of the Paper : C10-Ring Theory and Linear Algebra-I

Name of the Course : B.Sc. (H) Mathematics

Semester : IV

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *three* parts from each question.

All questions are compulsory.

1. (*a*) Let R be a ring. Prove that the center of R is a subring of R. Is Z_6 a subring of Z_{12} ? 3+2=5

(*b*) Show that a finite integral domain is a field. Give an example of the same. Also, give an example of an infinite integral domain which is not a field. 3+1+1=5

P.T.O.

(b) Using $v = \ln u$ and $v = f(x) + g(y)$, find the solution of the Cauchy problem

$$y^2 u_x^2 + x^2 u_y^2 = (xyu)^2, \quad \text{with } u(x,0) = e^{x^2}.$$ (7½)

(c) Find the integral surfaces of the equation $uu_x + u_y =$ for the following initial data : $x(s,0) = s$, $y(s,0) = 2$ $u(s,0) = s$. (7½)

SECTION – II

2. Attempt any **one** part out of the following :

(a) State Conservation Law and derive the Burgers Equation

$$u_t + uu_x = vu_{xx},$$

where v is the kinematic viscosity and u(x,t) is the fluid velocity field. (6

(b) Derive the damped wave equation of a string

$$u_{tt} + au_t = c^2 u_{xx},$$

where the damping force is proportional to the velocity and a is constant. Considering a restoring force proportional to the displacement of a string, show that the resulting equation is

$u_{tt} + au_t + bu = c^2 u_{xx}$, where b is a constant. (6

Attempt any **two** parts out of the following :

(a) Find the characteristics, characteristic coordinates and reduce the equation given below to the canonical form :

$u_{xx} + x^2 u_{yy} = 0, \quad \text{for } x \neq 0.$ (6)

(b) Use the polar coordinates r and θ ($x = r\cos\theta$, $y = r\sin\theta$) to transform the Laplace equation $u_{xx} + u_{yy} = 0$ into the polar form

$$\nabla^2 u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0.$$ (6)

(c) Obtain the general solution of the equation given below :

$x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} + xyu_x + y^2 u_y = 0$ (6)

SECTION – III

4. Attempt any **three** parts out of the following :

(a) Determine the solution of the Cauchy problem given below :

$u_{tt} - 9u_{xx} = 0, \; x \in R, \; t > 0$

$u(x,0) = \cos x, \qquad u_t(x,0) = \sin 2x$ (7)

(b) Find the solution of the initial boundary- value problem,

$u_{tt} = 4u_{xx}, \quad 0 < x < 1, \quad t > 0,$

$u(x,0) = 0, \quad 0 \le x \le 1,$

$u_t(x,0) = x(1-x), \quad 0 \le x \le 1,$

$u(0,t) = 0, \; u(1,t) = 0, \quad t \ge 0.$ (7)

(c) Solve the Cauchy problem for the non- homogeneous wave equation

$u_{tt} = c^2 u_{xx} + h^*(x,t),$

with the initial conditions

$u(x,0) = f(x), \qquad u_t(x,0) = g^*(x)$ (7)

(d) Solve the characteristic initial- value problem

$xu_{xx} - x^3 u_{yy} - u_x = 0, \; x \ne 0,$

$u(x,y) = f(y) \text{ on } y - \frac{x^2}{2} = 0 \text{ for } 0 \le y \le 2,$

$u(x,y) = g(y) \text{ on } y + \frac{x^2}{2} = 4 \text{ for } 2 \le y \le 4,$

where $f(2) = g(2)$. (7)

SECTION – IV

Attempt any **three** parts out of the following :

(a) Discuss the solution of the Heat Conduction Problem

$u_t = ku_{xx}, \ 0 < x < l, \ t > 0$

$u(0,t) = 0, \ t \geq 0$

$u(l,t) = 0, \ t \geq 0$

$u(x,0) = f(x), \ 0 \leq x \leq 1.$ (7)

(b) Solve using the method of separation of variables

$u_{tt} - c^2u_{xx} = 0, \quad 0 < x < \pi, \quad t > 0,$

$u(x,0) = \sin x, \quad u_t(x,0) = x^2 - \pi x, \quad 0 \leq x \leq \pi,$

$u(0,t) = u(\pi,t) = 0, \quad t > 0.$ (7)

(c) Determine the solution of

$u_{tt} = c^2u_{xx} + A \sinh x, \ 0 < x < l, \quad t > 0,$

$u,(x,0) = 0. \quad u_t(x,0) = 0, \quad 0 \leq x \leq l,$

$u(0,t) = h, \quad u(l,t) = k, \quad t > 0$

where h, k and A are constants. (7)

(d) Find the temperature distribution in a rod of length
The faces are insulated, and the initial temperature
distribution is given by $x(l - x)$.

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2815**

Unique Paper Code : **32351402** **GC-4**

Name of the Paper : **C-9, Riemann Integration and Series of Functions**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

1\. (*a*) Prove that a bounded function f on $[a, b]$ is integrable if and only if for each $\varepsilon > 0$, $\exists$ a $\delta > 0$ such that :

$$\text{mesh}(P) < \delta \Rightarrow U(f, P) - L(f, P) < \varepsilon,$$

$\forall$ partitions P of $[a, b]$. 6

(*b*) Show that every continuous function on $[a, b]$ is integrable. Is the converse true ? Justify. 6

(*c*) State and prove Intermediate Value Theorem for integrals. Show, with the help of an example, that the condition of continuity cannot be relaxed. 6

2. (*a*) (*i*) Let f be a bounded function on $[a, b]$. Define the Darboux Integral $\int_a^b f$. 3

(*ii*) Give an example of a function f which is not integrable, for which $|f|$ is integrable. 3

(*b*) (*i*) State Fundamental Theorem of Calculus-II. 2

(*ii*) Let f be a continuous function on **R**, and define :

$$F(x) = \int_{x-1}^{x+1} f(t)\, dt \quad \text{for } x \in \mathbf{R}$$

Show that F is differentiable on **R**, and compute F′. 4

(*c*) (*i*) Define :

$$f(x) = [x] \text{ on } [0, 3].$$

Show that f is integrable, and evaluate $\int_0^3 f(x)\, dx$. 3

(*ii*) Define :

$$g : [0, 2] \to [0, 4] \text{ as } g(x) = x^2.$$

Let :

$$P = \{0, \frac{1}{2}, 1, \frac{3}{2}, 2\}.$$

Find U(g, P). 3

(a) Show that :

$$\int_0^\infty e^{-t} t^{x-1}\, dt$$

converges $\Leftrightarrow x > 0$. 7

(b) Determine the convergence or divergence of :

$$\int_0^\infty \frac{1}{e^x \sqrt{x}}\, dx.$$ 7

(c) In the following, determine the values of r for which the integral converges/diverges. Determine the values of integrals in case of convergence.

(i) $\int_1^\infty x^{-r}\, dx$

(ii) $\int_0^\infty x^{-r}\, dx$. 7

(a) Let $\{f_n\}$ be a sequence of integrable functions on $[a, b]$ and suppose that $\{f_n\}$ converges uniformly on $[a, b]$ to f. Show that f is integrable. 6

(b) Let

$$f_n = \frac{1}{(1+x)^n}$$

for x in $[0, 1]$. Find the point-wise limit f of the sequence $\{f_n\}$ on $[0, 1]$.

Does $\{f_n\}$ converge uniformly to f on $[0, 1]$? Justify. 6

(c) Show that if $0 < b < 1$, then the convergence of the sequence :

$$f_n = \frac{x^n}{(1+x^n)},$$

for x in R, $x \geq 0$ is uniform on the interval $[0, b]$, but is not uniform on the interval $[0, 1]$. 6

5. (a) State and prove Weierstrass M-Test and hence examine the uniform convergence of the series :

$$\sum_{n=1}^{\infty} \frac{1}{(x^2 + n^2)}, \; x \text{ in } \mathbf{R}.$$ 6

(b) Show that the series of functions :

$$\sum \frac{1}{(1+x^n)}, \; x \neq 0,$$

converges uniformly on $[a, \infty)$ for $a > 1$, but is not uniformly convergent on $(1, \infty)$. 6

(c) Let $f : A \rightarrow \mathbf{R}$, $A \subseteq \mathbf{R}$ be a bounded function on A. Define uniform norm $\| f \|$ on A. State criterion for uniform convergence of a sequence of bounded functions using uniform norm and hence examine for uniform convergence the sequence :

$$f_n = x^n; \; x \in [0, 1].$$ 6

(a) (i) Given :

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

for $|x| < 1$. Evaluate $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$. 2.5

(ii) Let

$$f(x) = \sum a_n x^n$$

for $|x| < R$. If $f(x) = f(-x), \forall |x| < R$, then show that $a_n = 0$ for all odd values of n. 4

(b) (i) If the power series $\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence R, then show that series :

$$\sum_{n=1}^{\infty} n a_n x^{n-1} \text{ and } \sum_{n=0}^{\infty} \frac{a_n x^{n+1}}{n+1}$$

also have the radius of convergence R. 4

(ii) Show that :

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{for } x \in \mathbf{R}$$

is well-defined. Also show that : 2.5

$$f(x) = f'(x), \text{ for } x \in \mathbf{R}.$$

(c) (*i*) Define interval of convergence for a power series. Find the radius of convergence and interval of convergence for the following power series :

$$\sum_{n=1}^{\infty} \frac{x^n}{n}, \quad \sum_{n=1}^{\infty} \frac{x^n}{n^2}.$$

(*ii*) State Weierstrass's Approximation Theorem.

[This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 2816

Unique Paper Code : 32351403 GC-4

Name of the Paper : C10-Ring Theory and Linear Algebra-I

Name of the Course : B.Sc. (H) Mathematics

Semester : IV

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *three* parts from each question.

All questions are compulsory.

(*a*) Let R be a ring. Prove that the center of R is a subring of R. Is Z_6 a subring of Z_{12} ? 3+2=5

(*b*) Show that a finite integral domain is a field. Give an example of the same. Also, give an example of an infinite integral domain which is not a field. 3+1+1=5

P.T.O.

(*c*) Let n be an integer with decimal representation :

$$a_k\, a_{k-1} \dots a_1\, a_0$$

Prove that n is divisible by 9 if and only if :

$$a_k + a_{k-1} + \dots + a_1 + a_0$$

is divisible by 9. 5

(*d*) Show that a non-zero idempotent cannot be nilpotent. Find all the idempotent and nilpotent elements of the ring $\mathbf{Z}_4$. 2+3=5

2. (*a*) How many elements are in $R = \mathbf{Z}[i]/\langle 2 - i \rangle$. Give reasons for your answer. 5

(*b*) Let R be the ring of continuous functions from **R** to **R**. Show that :

$$A = \{f \in R \mid f(0) = 0\}$$

is a maximal ideal of R. 5

(*c*) Define a principal ideal domain and show that **Z** is a principal ideal domain. 1+4=5

(*d*) Determine all ring homomorphisms from $\mathbf{Z}_{12}$ to $\mathbf{Z}_{30}$. 5

(a) Prove that union of two subspaces of a vector space is a subspace if and only if one of the subspace is contained in the other. 5

(b) Show that for a subset W of a vector space V, W = span (W) if and only if W is a subspace of V. 5

(c) Check whether the subset :

S = {1, 1, 2, 4), (2, −1, −5, 2), (1, −1, −4, 0), (2, 1, 1, 6) of $\mathbf{R}^4$ is linearly dependent or linearly independent. 5

(d) Suppose that V be a vector space over a field F and let S = $\{x_1, x_2, ..., x_n\}$ be a linearly independent subset of V. Further, let $x \in$ V be such that $x \notin$ S then :

$$S_1 = \{x_1, x_1, x_2, ..., x_n\}$$

is linearly dependent if and only if $x \in$ span(S). 5

(a) Let S be a finite subset of vector space V such that V = span(S). Then, prove that there exists a subset T of S such that T is a basis of V. 5

(b) Let $\beta = \{v_1, v_2, ..., v_n\}$ be a subset of vector space V over F. Then, β forms a basis for V if and only if every element of V can be expressed uniquely as a linear combination of elements of β. 5

(c) Find bases for the following subspace of $\mathbf{R}^5$:

$W_1 = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbf{R}^5 : a_1 - a_3 - a_4 = 0\}$ and

$W_2 = \{(a_1, a_2, a_3, a_4, a_5) \in \mathbf{R}^5 : a_2 = a_3 = a_4$ and $a_1 + a_5 = 0\}$.

What are the dimensions of W_1 and W_2 ? 2,2,1

(d) Extend the set $S = \{1, 1, 0)\}$ to form two different bases of $\mathbf{R}^3$. 2.5,2.5

5. (a) Let V and W be vector spaces and $T : V \rightarrow W$ be a linear transformation.

If $\beta = \{v_1, v_2 ..., v_n\}$ is a basis for V, then prove that : 5

$$R(T) = \text{span}(T(\beta)) = \text{span}(\{T(v_1), T(v_2), .., T(v_n)\})$$

(b) Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^3$ be defined by :

$$T(x_1, x_2) = (x_1 - x_2, x_1, 2x_1 + x_2).$$

Let β be the standard ordered basis for $\mathbf{R}^2$ and :

$$\gamma = \{(1, 1, 0), (0, 1, 1), (2, 2, 3)\}$$

Compute $[T]_\beta^\gamma$. 5

(c) Let V, W and Z be vector spaces and let $T : V \to W$ and $U : W \to Z$ be linear transformations :

(i) Prove that if UT is one-to-one, then T is one-to-one.

(ii) Prove that if UT is onto, then U is onto. 2.5.,2.5

(d) Let :

$$A = \begin{pmatrix} 13 & 1 & 4 \\ 1 & 13 & 4 \\ 4 & 4 & 10 \end{pmatrix} \text{ and } \beta = \left\{ \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

be an ordered basis of $\mathbf{R}^3$. Then find $[L_A]_\beta$ and also find an invertible matrix Q such that $[L_A]_\beta = Q^{-1} AQ$, where L_A is a left-multiplication transformation. 5

[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2744 GC-4

Unique Paper Code : 32355444

Name of the Paper : Elements of Analysis

Name of the Course : **Mathematics : Generic Elective for Honours**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All questions are compulsory.

3. Attempt any **two** parts from each question.

1. (a) Define a denumerable set. Show that the set $\mathbb{Z}$ of all integers is denumerable. (7.5)

(b) Define supremum and infimum of a non empty subs of R. Find the supremum and infimum of each of th following sets.

(i) $A=\left\{1-\frac{(-1)^n}{n}: n \in N\right\}$

(ii) $B=\left\{\frac{1}{n}: n \in N\right\}$ (7.

(c) (i) For any two real numbers x and y prove that

$$|x+y| \leq |x|+|y|.$$

(ii) Define cluster point of a set $A \subseteq R$. Show th the finite set A = {1,2} has no cluster points.

(7.

2. (a) Use the definition of the limit of a sequence to sho that

(i) $\lim\limits_{n\to\infty}\left(\frac{3n+2}{n+1}\right)=3$

(ii) $\lim\limits_{n\to\infty}\left(\frac{1}{n^2+1}\right)=0$ (7.5

(b) State Monotone Convergence Theorem. Hence prove that the sequence $\langle x_n \rangle$ defined by

$$x_1 = 1,\ x_{n+1} = \sqrt{2x_n},\quad \text{for } n \in N$$

is convergent. (7.5)

(c) Define Cauchy sequence. Show that the sequence $\langle x_n \rangle = \langle 1 + (-1)^n \rangle$ is not a Cauchy sequence. (7.5)

(a) If the series $\sum_{n=1}^{\infty} a_n$ converges, then prove that $\lim_{n\to\infty} a_n = 0$. Is the converse true? Justify. (6.5)

(b) Show that the series $\sum_{n=1}^{\infty} r^n$ converges if and only if $|r| < 1$. (6.5)

(c) State limit comparison test for positive term series. Test the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{1}{n!}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ (6.

4. (a) Define an absolutely convergent and a conditiona convergent series. Prove that every absolute convergent series in R is also convergent.

(b) State Ratio test and hence prove that the series $\sum_{n=1}^{\infty}$ is convergent. (6

(c) Test the convergence and absolute convergence of t following series :

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+1}$

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

(a) Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(n!)^2 x^n}{(2n)!}.$$ (5)

(b) Derive the power series expansion for $f(x) = \sin x$. (5)

(c) Prove the identity

$$C(x+y) = C(x)C(y) - S(x)\,S(y), \text{ for all } x, y \in R$$

where $S(x)$ and $C(x)$ denote the sine and cosine functions respectively. (5)

(a) Determine the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$. Also check the convergence at the end points of the interval. (5)

(b) State integration theorem of power series. Show by integrating the series for $\frac{1}{1+x}$ that if $|x| < 1$, then

$$\log(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n.$$ (5)

(c) Define exponential function $E(x)$ as a sum of power series. Find the domain of $E(x)$ and show that

$$E(x+y) = E(x).E(y), \text{ for all } x, y \in R.$$

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1133**

Unique Paper Code : **235601** G

Name of the Paper : **Analysis-V**

Name of the Course : **B.Sc. (H) Mathematics–III**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are three sections in this question paper.

All questions are compulsory.

Attempt any *two* parts from each question.

Section I

1. (*a*) Let Σ be the sphere defined as :

$$\Sigma = \left\{(\xi, \eta, \zeta) \in \mathbf{R}^3 : \xi^2 + \eta^2 + \left(\zeta - \frac{1}{2}\right)^2 = \frac{1}{4}\right\}.$$

Let S be a circle on S and let T be the corresponding set in **C**. Prove that :

(*i*) T is a line if S contains (0, 0, 1) and

(*ii*) T is a circle if S does not contain (0, 0, 1). 6

P.T.O.

(*b*) Determine the set whose points satisfy the following relation : 3+3

(*i*) $\left|\dfrac{z-1}{z+1}\right| = 1$

(*ii*) $\left|Arg\ z - \dfrac{\pi}{2}\right| = \dfrac{\pi}{2}$.

Sketch these sets and determine which of these sets are region. Justify your answer.

(*c*) Find the radius of convergence of the following power series :

(*i*) $\sum_{n=0}^{\infty} [1+(-1)^n]^n z^n$ 2

(*ii*) $\sum_{n=1}^{\infty} \dfrac{z^n}{n}$ 2

(*iii*) $\sum_{n=1}^{\infty} \dfrac{n!\, z^n}{n^n}$ 2

2. (*a*) If the function f is differentiable at a point $z = x + iy$, prove that partial derivatives f_x and f_y exist at (x, y) and satisfy the Cauchy-Riemann equations $f_y = i f_x$. Is the converse true ? Justify your answer. 4+2

(b) Show that :

(i) the functions $f(z) = \bar{z}$ and $f(z) = \text{Re } z$ are not differentiable at any point of C.

(ii) the function $f(z) = x^2 + iy^2$ is differentiable at all points on the line $y = x$, but is nowhere analytic. 3+3

(c) Find all solutions of :

(i) $e^z = 1 + i$ and

(ii) $\sin z = 2$. 2+4

Section II

3\. (a) (i) Show that : 4

$$\int_C z^n \, dz = 0 \quad \text{if} \quad n \neq -1$$
$$= 2\pi i \quad \text{if} \quad n = -1$$

where C is the curve given by :

$$z(t) = \text{Re}^{it};\ 0 \leq t \leq 2\pi.$$

(ii) Evaluate : 2

$$\int_{|z|=1} \frac{\sin(z)}{z^3} \, dz.$$

(b) Suppose $C : z(t)\ ;\ t \in [a, b]$ is a smooth curve and f is a continuous complex function on C. If there exists an analytic function F on C such that $f(z) = F'(z)$, then prove that : 6

$$\int_c f(z)\, dz = F(z(b)) - F(z(a)).$$

(c) State and prove M-L formula. 6

4. (a) (i) Find the Taylor's series expansion of e^z about any point α. 4

(ii) Show that an odd entire function has only odd terms in its power series expansion about the origin. 2

(b) (i) State Cauchy Integral Formula for entire functions. 2

(ii) Suppose f is entire and $|f'(z)| \leq |z|,\ \forall z$. Show that :

$$f(z) = a + bz^2 \text{ with } |b| \leq \frac{1}{2}.$$ 4

(c) State and prove the Liouville's Theorem. 6

Section III

5\. (a) Suppose that a function f is analytic in a region D and that $f(z_n) = 0$ where (z_n) is a sequence of distinct points and $z_n \to z_0 \in D$. Then prove that $f \equiv 0$ in D. 7.5

(b) Find the Laurent expression for :

(i) $\dfrac{1}{z^2(1-z)}$ about $z = 0$ 2.5

(ii) $\dfrac{(z+1)^2}{z}$ about $z = 0$ 2.5

(iii) $\dfrac{1}{z^2-4}$ about $z = 2$ 2.5

(c) Show that the bilinear transformation :

$$f(z) = \frac{az+b}{cz+d}, \quad ad - bc \neq 0$$

is a 1-1 mapping of the Riemann sphere onto itself. 7.5

6\. (a) (i) Find the fundamental period of $\cos\dfrac{2\pi x}{k}$ and $\sin 2\pi x$. 1+1 (*ii*) Are the following functions $f(x)$, assuming these to be periodic of period 2π, even, odd or neither even nor odd ? 2

(1) $f(x) = x + x^2$

(2) $f(x) = \begin{cases} \cos\dfrac{\pi x}{l}, & 0 \le x \le \dfrac{l}{2} \\ 0 & , \dfrac{l}{2} \le x \le l \end{cases}$ 2

(*b*) Find the Fourier series of : 6

$$f(x) = \begin{cases} -k, & \text{if } -\pi < x < 0 \\ k, & \text{if } 0 < x < \pi \end{cases}.$$

(*c*) Find the Fourier cosine and sine series of the function : 6

$$f(x) = \begin{cases} \dfrac{2k}{L}x, & \text{if } 0 < x < \dfrac{L}{2} \\ \dfrac{2k}{L}(L - x), & \text{if } \dfrac{L}{2} < x < L \end{cases}$$

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 1134

Unique Paper Code : 235603 G

Name of the Paper : Algebra V

Name of the Course : B.Sc. (H) Mathematics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

Each part of questions 1, 2 and 3 carries 6 marks and each part of questions 4, 5 and 6 carries 6.5 marks.

1. (*a*) Determine the number of cyclic subgroups of order 10 in $Z_{100} \oplus Z_{25}$.

(*b*) Show that D_4 (the Dihedral group of order 8) cannot be written as an internal direct product of two of its proper subgroups.

(c) Let G and H be finite cyclic groups. Prove that $G \oplus H$ is cyclic if and only if :

$$\gcd(|G|, |H|) = 1.$$

2. (a) Let G be a group acting on itself by left multiplication. Find kernel of this action. Is it faithful ? Is it transitive ?

(b) For $G = S_3$ and a subgroup A of G such that :

$$A = \{1, (123), (132)\},$$

prove that :

$$C_G(A) = A \text{ and } N_G(A) = G.$$

(c) If G is a finite group and p is the smallest prime dividing $|G|$, then any subgroup of index p is normal.

3. (a) State the class equation for a finite group. Verify it for Q_8 (Quaternion group) and D_8 (the Dihedral group of order 8).

(b) Prove that the two elements of S_n are conjugate if and only if they have the same cycle type.

(c) Let G be a group of order 12. Show that either G has a normal Sylow 3-subgroup or G is isomorphic to A_4.

4. (a) State and prove the index theorem for a finite group. Hence or otherwise show that there is no simple group of order 216.

(b) Let T be a self adjoint operator on a finite dimensional inner product space V. Prove that for all :

$$x \in V, \| T(x) \pm ix \|^2 = \| T(x) \|^2 + \| x \|^2$$

Deduce that $T - iI$ is invertible and that :

$$[(T - iI)^{-1}]^* = (T + iI)^{-1}.$$

(c) Let T be an orthogonal operator on $\mathbf{R}^2$ and let $A = [T]_\beta$, where β is the standard ordered basis for R^2. Show that exactly one of the following conditions is satisfied :

(i) T is a rotation and det (A) = 1.

(ii) T is a reflection about a line through the origin and det(A) = −1.

5. (*a*) Find new coordinates X and Y so that the quadratic form : $x^2 - 2xy + y^2$ can be written as :

$$\lambda_1 X^2 + \lambda_2 Y^2.$$

(*b*) If V is the space of all polynomials of degree less than or equal to *n* over a field F, prove that the differentiation operator on V is nilpotent.

(*c*) Let T be the linear operator on $\mathbf{R}^3$ which is represented in the standard ordered basis by the matrix :

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

Let W be the null space of T − 2I. Prove that W has no complementary T-invariant subspace.

6. (*a*) Let T be a linear operator on $\mathbf{R}^3$ which is represented in the standard ordered basis by the matrix :

$$\begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}.$$

(i) Express the minimal polynomial p for T in the form $p = p_1 p_2$, where p_1 and p_2 are monic irreducible over the field of real numbers.

(ii) Let W_i be the null space of $p_i(T)$. Find bases β_i for the spaces W_1 and W_2. If T_i is the operator induced on W_i by T, find the matrix of T_i in the basis β_i.

(b) Define Jordan form of a square matrix. If A is a complex 5×5 matrix with characteristic polynomial :

$$f(x) = (x-2)^3 (x+7)^2,$$

find all the possible Jordan forms for A.

(c) Let V be an n-dimensional vector space, and let T be a linear operator on V. Suppose that T is diagonalizable :

(i) If T has a cyclic vector, show that T has n distinct characteristic values.

(*ii*) If T has n distinct characteristic values, and if

$$\{v_1, v_2, \ldots\ldots, v_n\}$$

is a basis of characteristic vector for T, show that :

$$v = v_1 + v_2 + \ldots\ldots + v_n$$

is a cyclic vector for T.

This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1135**

Unique Paper Code : **235604** **G**

Name of the Paper : **Discrete Mathematics**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do any *two* parts from each question.

Section I

1. (*a*) Prove that two finite ordered sets P and Q are order isomorphic if and only if they can be drawn with the identical diagrams. 6

(*b*) Draw the diagrams of ordered sets

$$2\times 3,\ \overline{2}\cup 3,\ 1\oplus\overline{3}\oplus 1$$

where n denotes the chain $0<1<....<n-1$ and $\overline{n}$ is the anti-chain for the set $\{0, 1, ... n-1\}$. 6

P.T.O.

(c) Find if the mapping $\theta : P \to Q$ is order preserving for $P = \langle \wp(S), \subseteq \rangle$ with $|S| > 1, Q = 2$ and $\theta(U) = 1$ if $U = S$ and $\theta(U) = 0$ if $U \neq S$ where $\wp(S)$ is the power set of S consisting of all subsets of S. 6

2. (a) Let $(L, \vee, \wedge)$ be a non-empty set equipped with two binary operations which satisfy associative, commutative, idempotency and absorption laws. Define a relation '$\le$' on L by $a \le b$ if and only if $a \wedge b = a$. Prove that $(L, \le)$ is a lattice ordered set. 6½

(b) If '$\le$' is defined on any subset of $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ by $m \le n$ if and only if m divides n then show that the ordered subset

$$Q = \{1, 2, 4, 5, 6, 12, 20, 30, 60\}$$

of $(\mathbb{N}_0, \le)$ does not form a lattice. Draw a diagram of Q and find elements $a, b, c, d \in Q$ such that $a \vee b$ and $c \wedge d$ do not exist in Q. 6½

(c) Let L be a lattice. Prove that the following are equivalent :

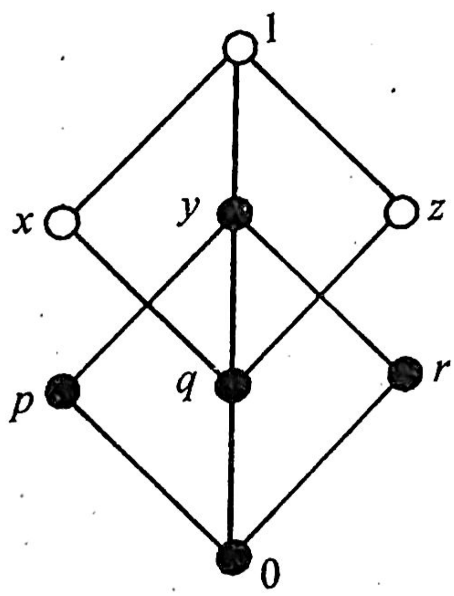
(i) L is a chain.

(ii) Every non-empty subset of L is a sublattice.

(iii) Every two element subset of L is a sublattice. 6½

Section II

3. (a) Define modular and distributive lattices. Use $M_3 - N_5$ theorem to find if the following lattice is modular or distributive. 6



(b) Find the disjunctive normal form of : 6

$$f(x_1, x_2, x_3) = x_1(x_2 + x_3)' + (x_1x_2 + x_3)x_1 .$$

(c) Find all the prime implicants of :

$$xy'z + x'yz' + xyz' + xyz$$

and form the corresponding prime implicant table. 6

4. (a) (i) In a Boolean algebra B, show that $\forall\, x, y \in B$

$$x \leq y \Leftrightarrow x \wedge y' = 0 \Leftrightarrow x' \vee y = 1 \Leftrightarrow x \wedge y$$

$$= x \Leftrightarrow x \vee y = y$$ 3½

(ii) Show that (B, gcd, lcm) is a Boolean algebra for the set B of all positive divisors of 30. 3

(b) Use a Karnaugh map diagram to simplify :

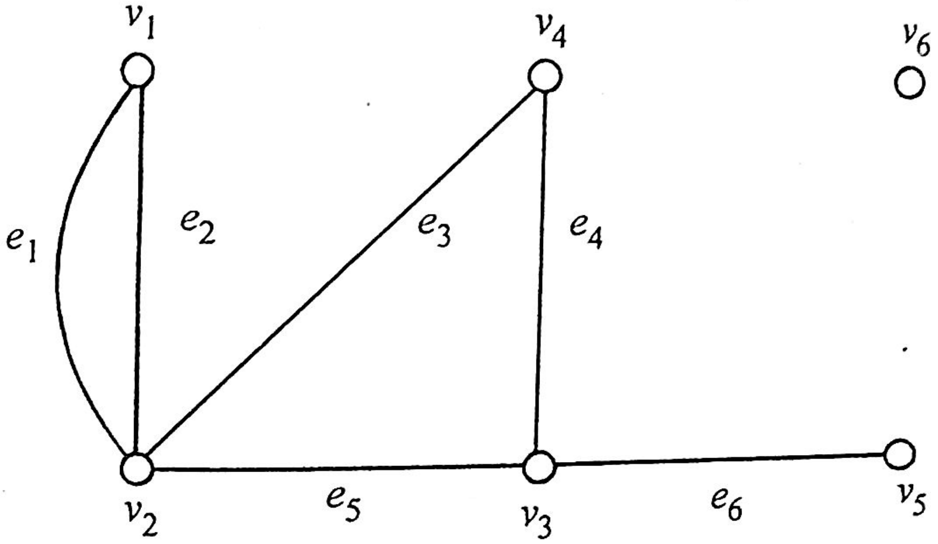
$$(x_1 + x_2)(x_1 + x_3) + x_1x_2x_3 .$$ 6½

(c) Draw the symbolic representation of the circuit given by :

$$p = (x_1 + x_3)'(x_1' + (x_2 + x_3)(x_2' + x_3')) .$$ 6½

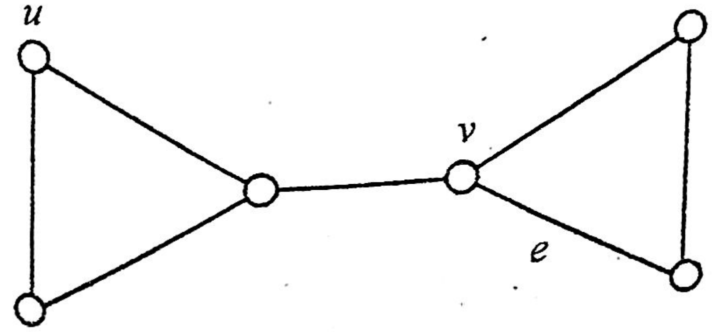
Section III

(a) (i) Find the degree sequence for G. Verify that the sum of the degrees of the vertices is an even number ? Which vertices are even ? Which are odd ? 3

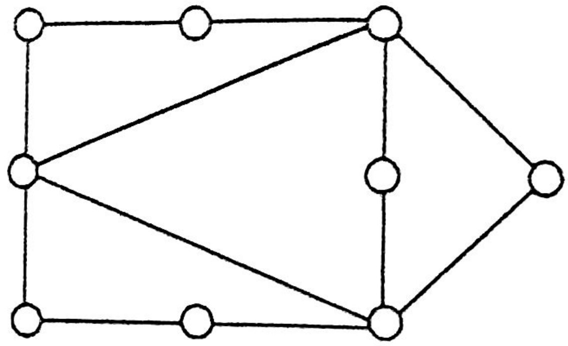


G

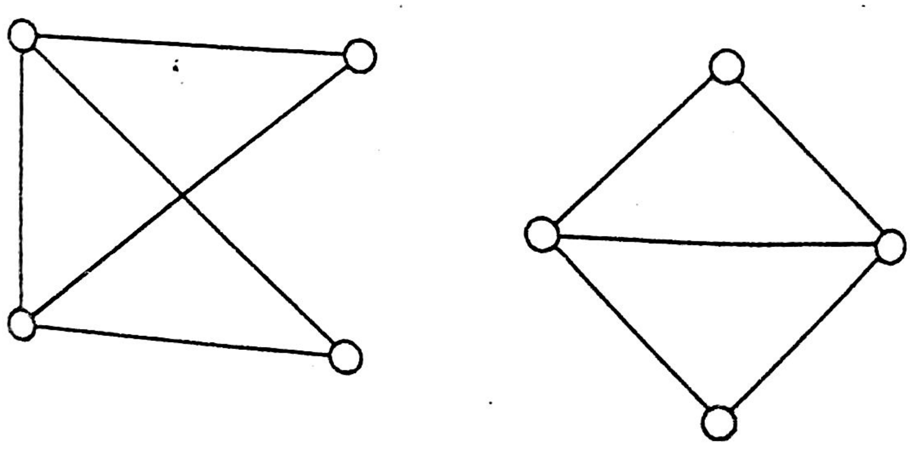
(ii) Define a subgraph of a graph. Draw pictures of the subgraphs $G \setminus \{e\}, G \setminus \{v\}, G \setminus \{u\}$ where G is : 3



(b) (i) Determine whether the graph below is bipartite. Give the bipartition sets or explain why the graph is not bipartite. 3



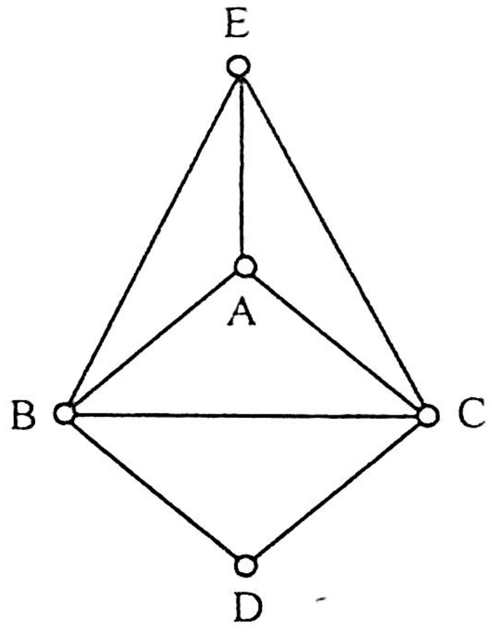
(ii) Define isomorphism of graphs. Also label the graphs so as to show an isomorphism. 3



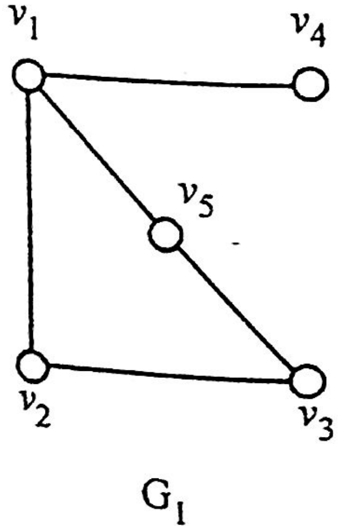
(c) (i) Define a complete graph. Does there exist a graph G with 28 edges and 12 vertices each of degree 3 or 4. Justify your answer. 3

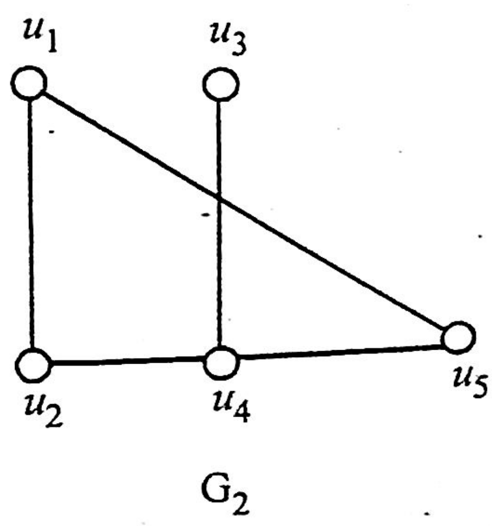
(ii) Why can't there exist a graph whose degree sequence is 5, 4, 4, 3, 2, 1 ? Draw a graph with degree sequence 4, 3, 2, 2, 1. 3

(a) Define a Hamiltonian graph. Is the following graph Hamiltonian ? Is it Euclidean ? Explain your answer. 6½

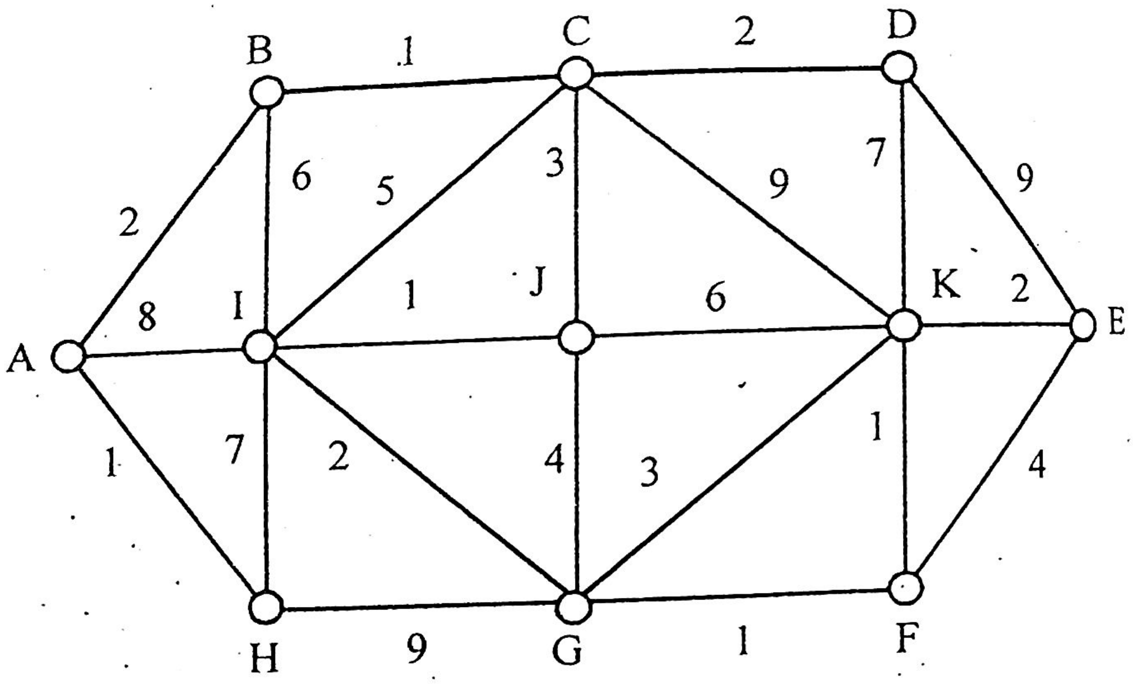


(b) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$. 6½





(c) Apply the original version of Dijkstra's algorithm to find a shortest path from A to F. Write steps. 6½



This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1138**

Unique Paper Code : **235607** **G**

Name of the Paper : **Number Theory**

Name of the Course : **B.Sc. (Honors) Mathematics**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

Question Nos. **1** to **5** each part carries **6½** marks

and in question **6** each part carries **5** marks.

1. (*a*) Determine all solutions in the integers of the Diophantine equations :

$$56x + 72y = 40.$$

(*b*) Find the remainder obtained upon dividing the sum :

$$1! + 2! + 3! + \dots\dots + 100!$$

by 12.

(*c*) Solve the linear congruence :

$$17x \equiv 9 \bmod (276).$$

2\. (*a*) State and prove Wilson's theorem.

(*b*) If p and q are distinct primes, prove that :

$$p^{q-1} + q^{p-1} \equiv 1 (\bmod\ pq).$$

(*c*) Using Wilson's theorem prove that for any odd prime p :

$$1^2 . 3^2 . 5^2 \dots\dots (p-2)^2 \equiv (-1)^{\frac{(p-1)}{2}} (\bmod\ p).$$

3\. (*a*) Let r be a primitive root of the odd prime P. Then prove the following :

(*i*) If $p \equiv 1 \pmod 4$, then $-r$ is also a primitive root of p.

(*ii*) If $p \equiv 3 \pmod 4$, then $-r$ has order $(p-1)/2$.

(b) If $a_1, a_2, \ldots\ldots, a_{\Phi(n)}$ is a reduced set of residues modulo n, show that :

$$a_1 + a_2 + \ldots\ldots + a_{\Phi(n)} \equiv 0 \pmod{n}, \text{ for } n > 2.$$

(c) For $k \geq 3$, show that the integer 2^k has no primitive roots.

(a) State and prove Euler's theorem.

(b) Find the order of 3 and 5 modulo 17.

(c) Define Euler's Phi-function. For $n > 2$, show that $\Phi(n)$ is an even integer.

(a) Evaluate the following Legendre symbols (71/73), (29/53).

(b) Find quadratic residues of 13.

(c) Show that 3 is a quadratic residue of 23, but a non-residue of 31.

(a) Using the linear cipher :

$$C \equiv 5P + 11 \pmod{26},$$

encrypt the message CRYPTOGRAPHY.

(*b*) Use the Hill's cipher

$C_1 \equiv 2P_1 + 3P_2 \pmod{26}$

$C_2 \equiv 5P_1 + 8P_2 \pmod{26}$

to encrypt the message BUY NOW.

(*c*) Encrypt the message REUTRN HOME using the

Cipher.

(5)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1687

F-8

Unique Paper Code : 2351601

Name of the Paper : Algebra V (Ring Theory & Linear Algebra – II)

Name of the Course : B.Sc. (H) Mathematics

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) If D is an integral domain, then prove that D[x] is an integral domain.

   (b) Let R be a commutative ring. Show that R[x] has a subring isomorphic to R.

   (c) Let $f(x) = x^3 + 2x + 4$ and $g(x) = 3x + 2$ in $\mathbb{Z}_5[x]$. Determine the quotient and remainder upon dividing f(x) by g(x). (6,6,6)

(c) Is $A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix}$ diagonalizable ? (6½,6½,6½)

5. (a) Prove that in an inner product space V, $||x + y|| \leq ||x|| + ||y||$ for all $x, y \in V$: Verify the above inequality for $x = (2, 1 + i, i)$ and $y = (2 - i, 2, 1 + 2i) \in \mathbb{C}^3$.

(b) Let V be an inner product space. Let S be orthogonal subset of V consisting of nonzero vectors. Prove that S is linearly independent.

(c) Let W_1 and W_2 be subspaces of finite dimensional inner product space V. Prove that

(i) $(W_1 + W_2)^{\perp} = W_1^{\perp} \cap W_2^{\perp}$

(ii) $(W_1 \cap W_2)^{\perp} = W_1^{\perp} + W_2^{\perp}$ (6,6,6)

6. (a) For the data {(–1, 1), (0, 2), (1, 2), (2,5)}, find a linear function of best fit using the least square approximation. Also, find the error E.

(b) Let T be a linear operator on a finite dimensional complex inner product space V. Prove that T is normal ⇔ there exists an orthonormal basis for V consisting of eigen vectors of T.

2. (a) State and prove Gauss's lemma.

(b) Show that $1 - i$ is irreducible in $\mathbb{Z}[i]$

(c) In an Integral Domain, prove that e
is an irreducible element.

3. (a) For the function f defined on a vector
if f is a linear functional where

$V = \mathbb{R}^2;\ f(x, y) = (2x, 4y).$

(b) For the vector space V and its basi
formula for vectors of the dual basis
$V = \mathbb{R}^3;\ \beta = \{(1,0,1), (1,2,1), (0,0,1)\}$

(c) Define $f \in (\mathbb{R}^2)^*$ by $f(x, y) = 2x + y$
by $T(x, y) = (3x + 2y, x)$. Computer T

4. (a) Let T be the linear operator on P
$T(f(x)) = f'(x)$. Let W be the T-
generated by x^2. Is W a T-invariant su

(b) Let T be a linear operator on a finite di
space V. Let p(t) be the minimal polyno
that λ is an eigen value of T if and o
Hence, show that the characteristic pol
minimal polynomial of T have the same

(c) Let V be a finite dimensional in
over F and $g : V \to F$ be a linea
prove that there exists a unique vec
$g(x) = \langle x, y \rangle \ \forall \ x \in V$.

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 6514

Unique Paper Code : 231683 G

Name of the Paper : Culture in India : Ancient (A Historical Perspective)

Name of the Course : B.Sc. (H) Mathematics

Semester : VI

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

(इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए ।)

Note :— Answers may be written *either* in English *or* in Hindi; but the same medium should be used throughout the paper.

टिप्पणी : इस प्रश्न-पत्र का उत्तर अंग्रेज़ी या हिन्दी किसी एक भाषा में दीजिए; लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए ।

Attempt *four* questions in all.

All questions carry equal marks.

कुल चार प्रश्न कीजिये ।

सभी प्रश्नों के अंक समान हैं ।

P.T.O.

1. What light do the terracotta figuriness from Mohenjodaro throw on the Harappan civilization ?

मोहनजोदड़ो से प्राप्त मृण्मूर्तियाँ हड़प्पा सभ्यता पर क्या प्रकाश डालती हैं ?

2. Analyse the memorial stones found from different parts of South India.

दक्षिण भारत के विभिन्न क्षेत्रों से प्राप्त स्मारक पत्थरों का विश्लेषण करें ।

3. Discuss the different types of male and female characters as depicted in Bharatamuni's Natyashastra.

भरतमुनि रचित नाट्यशास्त्र में चित्रित विभिन्न प्रकार के पुरुष एवं महिला चरित्रों का वर्णन कीजिए ।

4. How women have been portrayed in Kalidasa's Abhijnan-shakuntalam ?

कालिदास कृत अभिज्ञानशाकुन्तलम में स्त्रियों को कैसे चित्रित किया गया है ?

5. Karna could not prove himself to be a good friend or a good warrior. Elaborate.

कर्ण अपने आपको न एक अच्छा मित्र, न एक अच्छा योद्धा सिद्ध कर सका । विस्तार कीजिए ।

6. Describe women and patriarchy in the Tamil fold songs.

तमिल लोकगीतों में महिलाओं एवं पितृसत्ता का वर्णन कीजिए ।

7. Write an essay on 'Pathur Nataraja'.

'पाथुर नटराज' पर एक निबंध लिखिए ।

8. Write short notes on any *two* :

(*a*) Meghdutam

(*b*) Jataka literature

(*c*) Vishwakarma Craftsmen of medieval South India

(*d*) Sangam Literature.

किन्हीं दो पर संक्षिप्त टिप्पणी लिखिए :

(अ) मेघदूतम्

(ब) जातक कथाएँ

(स) मध्यकालीन दक्षिण भारत में विश्वकर्मा शिल्पकार

(द) संगम साहित्य ।

This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6514A**

Unique Paper Code : **231684** **G**

Name of the Paper : **History-1 B : Culture in India-Medieval**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :— Answers may be written *either* in English *or* in Hindi; but the same medium should be used throughout the paper.

टिप्पणी : इस प्रश्न-पत्र का उत्तर अंग्रेज़ी **या** हिन्दी किसी एक भाषा में दीजिए; लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए।

Attempt any *four* questions.

All questions carry equal marks.

किन्हीं **चार** प्रश्नों के उत्तर दीजिए।

सभी प्रश्नों के अंक समान हैं।

P.T.O.

1. What was the nature of relationship between the king and the clown in south Indian myth and poetry ?

दक्षिण भारत के प्रचलित मिथकों और कविताओं में राजा विदूषक के सम्बन्ध किस प्रकार दर्शाए गए हैं ?

2. How is popular Shiaism reflected in Awadh ? Is it similar to the way it is practised in Iran ?

अवध में लोकप्रिय शिया धर्म किस प्रकार दर्शाया गया है ? क्या यह ईरान में भी प्रचलित है ?

3. Bhakti was not a part of the 'lower-class' ethos in medieval South India. Do you agree ?

मध्यकालीन दक्षिण भारत में भक्ति केवल 'निम्न जातियों' का ही जीवन मूल्य नहीं था। क्या आप सहमत हैं ?

4. Explain with examples how gender bias is reflected in Urdu ghazal. What is the basic difference between *rekhta* and *rekhti* ?

उर्दू गजल में लिंग भेद कैसे चित्रित किया जाता है ? उदाहरण देकर समझाइए। रेख्ता और रेख्ती में मूलभूत अंतर बताइए।

5. In what ways is the *guru-shishya parampara* reflected in the *adab* of musicians ?

संगीतज्ञों के अदब में गुरु-शिष्य परंपरा किस प्रकार प्रतिबिंबित होती है ?

6. Discuss the theme of love in Qutban's Mrigawati.

कुतबन की मृगावती में प्रेम की अवधारणा की व्याख्या कीजिए।

7. Discuss the salient features of Mughal painting during the region of Shah Jahan.

शाहजहाँ के शासनकाल में मुगल चित्रकला के प्रमुख पहलुओं का विवेचन कीजिए।

8. Write short notes on any *two* of the following :

(*a*) Birbal

(*b*) Alwar Saints

(*c*) 'Others' in Sanskrit literature

(*d*) Portuguese in Calicut.

निम्नलिखित में से किन्हीं **दो** पर संक्षिप्त टिप्पणी लिखिए :

(*a*) बीरबल

(*b*) अलवार संत

(*c*) संस्कृत साहित्य में 'दूसरे'

(*d*) पुर्तगाली और कालीकट।

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6514B**

Unique Paper Code : **231685** **G**

Name of the Paper : **Culture in India : Modern**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

(इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए ।)

Note :— Answers may be written *either* in English *or* in Hindi; but the same medium should be used throughout the paper.

टिप्पणी :—इस प्रश्न-पत्र का उत्तर अंग्रेज़ी या हिन्दी किसी एक भाषा में दीजिए; लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए।

Attempt any *four* questions.

All questions carry equal marks.

किन्हीं चार प्रश्नों के उत्तर दीजिये।

सभी प्रश्नों के अंक समान हैं।

P.T.O.

1. Discuss the various issues related to the game of cricket during the colonial times.

   औपनिवेशिक काल के दौरान भारत में क्रिकेट के खेल से जुड़े विभिन्न विषयों की विवेचना कीजिए।

2. Discuss the themes of some of the popular films of the 1940s and 1950s.

   1940 और 1950 के दशकों में कुछ लोकप्रिय फिल्मों के विषयों की विवेचना कीजिए।

3. In what ways is the 'National Museum' important to the Indian nation ?

   'राष्ट्रीय संग्रहालय' भारतीय राष्ट्र के लिए किस प्रकार महत्वपूर्ण है ?

4. What does 'Amar Jiban' tell us about the lives of Indian women in the 19th century ?

   'आमार जीबन' 19वीं सदी की भारतीय महिलाओं के जीवन के संबंध में क्या बताती है ?

5. Discuss the significance of oral traditions in formulating a sense of our cultural past ?

   हमारे सांस्कृतिक अतीत के प्रति बोध-निर्माण में मौखिक परंपराओं के महत्व की विवेचना कीजिए।

6. Discuss how traditional Indian music has changed in an encounter with modern technologies ?

परंपरागत भारतीय संगीत आधुनिक तकनीकों के प्रभाव से किस प्रकार प्रभावित हुआ ? विवेचना कीजिए।

7. What have been the themes relevant to nation-building that have been expressed in calendar art ?

कैलेंडर कला में अभिव्यक्त कौनसी विषयवस्तु राष्ट्र-निर्माण के लिए प्रासंगिक थी ?

300

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2002**

Unique Paper Code : **62351201** **GC-4**

Name of the Paper : **Algebra**

Name of the Course : **B.A. (Prog.) Discipline Course**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Prove that the set V of all ordered triples of real numbers of the form $(x, y, 0)$ under the operations $\oplus$ and $\odot$ defined by :

$$(x, y, 0) \oplus (x', y', 0) = (x + x', y + y', 0)$$

$$c \odot (x, y, 0) = (cx, cy, 0)$$

forms a vector space over **R**. 6

P.T.O.

2002

(*b*) Let V be a vector space with operators $\oplus$ and $\odot$. Let W be a non-empty subset of V. Prove that W is a subspace of V if and only if the following conditions hold :

(*i*) If u, v are vectors in W, then $u \oplus v$ is in W

(*ii*) If c is any real number and u is any vector in W, then $c \odot u$ is in W. 6

(*c*) Define basis of a vector space V. Check whether the set $\{(3, 2, 2), (-1, 2, 1), (0, 1, 0)\}$ forms basis for R^3 ? 6

2. (*a*) Reduce the matrix :

$$\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$

to its normal form and hence determine its rank. 6½

(*b*) Verify that the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

satisfies its characteristic equation and hence obtain A^{-1}. 6½

(c) Solve the system of linear equations : 6½

$$x - 3y + z = -1$$

$$2x + y - 4z = -1$$

$$6x - 7y + 8z = 7.$$

3. (a) If

$$\cos\alpha + 2\cos\beta + 3\cos\gamma = 0 = \sin\alpha + 2\sin\beta + 3\sin\gamma,$$

prove that :

$$\cos 3\alpha + 8\cos 3\beta + 27\cos 3\gamma = 18\cos(\alpha + \beta + \gamma)$$

$$\sin 3\alpha + 8\sin 3\beta + 27\sin 3\gamma = 18\sin(\alpha + \beta + \gamma).$$ 6

(b) Prove that : 6

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + \tan^4\theta}.$$

(c) Sum the series :

$$\cos\theta\sin\theta + \cos^2\theta\sin 2\theta + \ldots\ldots + \cos^n\theta\sin n\theta,$$

where $\theta \neq k\pi$. 6

4. (*a*) Find the rational roots of the equation : 6

$$x^4 - x^3 - 19x^2 + 49x - 30 = 0.$$

(*b*) Solve the equation :

$$27x^3 + 42x^2 - 28x - 8 = 0.$$

the roots being in G.P. 6

(*c*) If α, β, γ, be the roots of the equation : 6

$$x^3 + px^2 + qx + r = 0, \ (r \neq 0).$$

find the value of :

(*i*) $\Sigma(\beta + \gamma)^2$

(*ii*) $\Sigma\alpha^{-2}$.

5. (*a*) Let n be a positive integer. Prove that the congruence class $[a]_n$ has a multiplicative inverse in Z_n if and only if $(a, n) = 1$. 6½

(b) Consider the following permutations in S_7 : 6½

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 6 & 1 & 7 \end{pmatrix}$$

and $$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 7 & 4 & 6 & 3 \end{pmatrix}$$

Compute the following products :

(i) $\tau^{-1}\sigma\tau$

(ii) $\tau^2\sigma$.

(c) Prove that the set Q^+ of all positive rational numbers is an abelian group under the binary operation * defined by $a * b = \dfrac{ab}{3}$. 6½

6. (a) Let $G = GL_2(R)$. Prove that :

$$D = \left\{ \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}, ad \neq 0 \right\}$$

is a subgroup of G. 6

(*b*) Prove that the set S = {0, 2, 4} is a subring of the ring Z_6 of integers modulo 6.

(*c*) Prove that the rigid motions of an equilateral triangle yields the group S_3.

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : 3019

Unique Paper Code : 62354443

GC-4

Name of the Paper : Analysis

Name of the Course : B.A. (Prog.) Mathematics

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has *six* questions in all.

Attempt any *two* parts from each question.

All questions are compulsory.

Marks are indicated against each part of the questions.

1. (*a*) Which of the following sets are bounded below, which are bounded above and which are neither bounded below nor bounded above.

(*i*) $\{-1, -2, -3, -4, \ldots\ldots\ldots, -n, \ldots\ldots\}$

(*ii*) $\{-1, 2, -3, 4, \ldots\ldots\ldots, (-1)^n n, \ldots\ldots\}$

(*iii*) $\{2, \frac{3}{2}, \frac{4}{3}, \ldots\ldots\ldots, \left(\frac{n+1}{n}\right), \ldots\ldots\}$

(*iv*) $\{3, 3^2, 3^3, \ldots\ldots\ldots, 3^n, \ldots\ldots\}$

(v) $\{1, \frac{1}{2}, \frac{1}{2^2}, \ldots\ldots\ldots, \frac{1}{2^n} \ldots\ldots\}$.

(vi) $\{-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \ldots\ldots\ldots, \frac{(-1)^n}{2}, \ldots\ldots\}$ 6.5

(b) Define supremum and infimum of a non-empty bounded set. Suppose A and B are two non-empty subsets of **R** satisfying the property :

$$a \leq b, \forall a \in A \text{ and } \forall b \in B.$$

Prove that :

$$\sup(A) \leq \inf(B).$$

6.5

(c) State Bolzano Weirstrass theorem for sets. Show by an example that the conditions in this theorem cannot be relaxed. 6.5

2. (a) Prove that every continuous function on a closed interval is bounded. 6.5

(b) Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $]0, \infty[$. 6.5

(c) Define an open set and prove that the union of an arbitrary family of open sets is an open set. 6.5

3. (*a*) Define a convergent sequence and a bounded sequence. Show that the sequence $< a_n >$ defined by :

$$a_n = (-1)^n, \forall n$$

is bounded but not convergent. 6.5

(*b*) Show that the sequence $< a_n >$ defined by :

$$a_1 = 1, \quad a_{n+1} = \sqrt{2 + a_n}, \ \forall \ n \geq 1$$

is bounded and monotonic. Also find $\lim\limits_{n \to \infty} a_n$. 6.5

(*c*) State and prove Cauchy convergence criterion for sequences. 6.5

4. (*a*) Prove that every monotonically increasing and bounded above sequence converges. 6

(*b*) If $< a_n >$ and $< b_n >$ are sequences of real numbers such that :

$$\lim_{n \to \infty} a_n = a, \quad \lim_{n \to \infty} b_n = b$$

then prove that :

$$\lim_{n \to \infty} (a_n + b_n) = (a + b).$$ 6

(*c*) Show that :

$$\lim_{n \to \infty} \frac{2^{3n}}{3^{2n}} = 0.$$ 6

5. (a) State and prove Cauchy's nth root test for an infinite series. 6

(b) Test for convergence the following series :

(i) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+5}$

(ii) $\sum_{n=1}^{\infty} 2^{-n-(-1)^n}$. 6

(c) State (without proof) D'Alambert's ratio test for an infinite series. Test for convergence the series :

$$\frac{1}{5}+\frac{2!}{5^2}+\frac{3!}{5^3}+\frac{4!}{5^4}+\ldots\ldots\ldots\ldots.$$ 6

6. (a) Define an alternating series. State Leibnitz's test for an alternating series. Test for convergence the series :

$$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{6}+\ldots\ldots\ldots\ldots.$$ 6

(b) Test for convergence the following series :

(i) $\sum_{n=1}^{\infty} \frac{(-1)^n \sin n\alpha}{n^3}$, α being real

(ii) $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}$. 6

(c) Prove that every monotonically increasing function f on $[a, b]$ is Riemann integrable on $[a, b]$. 6

[This question paper contains 14 printed pages.]

Your Roll No. ....

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B.A. Prog. / III G

Paper Code : C-155

MATHEMATICS – Paper III

(Selected Topics in Mathematics)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Note :– The maximum marks printed on the question paper are applicable for the students of the regular colleges (Cat. A). These marks will, however, be scaled up proportionately in respect of the students of NCWEB at the time of posting of awards for compilation of result.

Attempt **six** *questions in all selecting* **two** *parts from each question.* **Unit I** *and* **Unit II** *are compulsory and contain* **four** *questions. In* **Unit III** *choose* **one** *of* **III(1), III(2), ..., III(5)** *and attempt* **both** *questions from the same. Marks are indicated against each question.*

Use of scientific calculator is allowed.

P.T.O.

Unit I

(Real Analysis)

1. (a) Prove that the intersection of a finite number of open sets is open. What happens if the family consists of infinite number of open sets ? Justify your answer. (6)

(b) Define limit point of a set. Prove that a finite set has no limit point. (6)

(c) Discuss the continuity at $x = 3$ of the function f defined by

$f(x) = x - [x] \ \forall \ x \geq 0$, where $[x]$ is the greatest integer $\leq x$. (6)

2. (a) Prove that sequence $<a_n>$ defined by the relation

$$a_1 = 1,\ a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \text{..........} + \frac{1}{(n-1)!}\ (n \geq 2),$$

converges. (7)

(b) Test the convergence of any two of the following series :

(i) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^p}$

(ii) $1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+\ldots\ldots\ldots$

(iii) $\sum\frac{\sqrt{n+1}-\sqrt{n-1}}{n}$ (7)

(c) State Leibnitz Test for the convergence of an alternating series. Test the convergence and the absolute convergence of the series :

$\frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}+\ldots\ldots\ldots$ (7)

3. (a) Prove that the function f(x) defined on [0,1] as :

$$f(x)=\begin{cases}1 & \text{if x is rational}\\ 0 & \text{if x is irrational}\end{cases}$$

is not Riemann integrable. (6)

(b) Show that the integral

$$\int_0^\infty e^{-x}x^{n-1}\,dx$$

converges if and only if $n>0$. (6)

(c) (i) Prove that the series

$$\frac{\sin x}{\sqrt{1}}+\frac{\cos 2x}{\sqrt{2}}+\frac{\sin 3x}{\sqrt{3}}+\frac{\cos 4x}{\sqrt{4}}+\ldots\ldots\ldots\ldots$$

is not a Fourier series.

(ii) Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

(6

Unit II

(Computer programming)

4. (a) (1) What are the two forms of floating constants in C/C++ ?

(2) What will be the value of 15.9/3 and 19% 6 in C/C++ ?

(3) How do the computer detects whether an identifier used in a program is a constant or a variable or an array ? Support your answer with example, for all three. (6½)

(b) (1) What will be the value of b if a = 10 initially when the following statement is executed ?

1. b = ++a + ++a & 2. b = ++a + a++ (6½)

(2) Give short notes on the iterative structures in C/C++, with examples. (6½)

(c) Write a program to calculate the area of a circle, a rectangle & a triangle depending upon user's choice, using switch structure. (6½)

Unit III(1)

(Numerical Analysis)

(a) Consider the equation

$$f(x) = x^4 - 3x^2 + x - 10 = 0$$

(1) Find the interval of unit length which contains smallest positive root of the equation.

(2) Perform two iterations by the Bisection Method, taking the initial interval as considered in part (1).

(3) Taking the midpoint of the last interval of part (2) as the initial approximation, obtain the root, correct to two places of decimal, by the Newton Raphson Method. (6)

(b) Compare the Bisection method with Newton Raphson method for solving an equation.

Also do the comparison of Gauss Seidel Method with the Gauss Jacobi method, for solving the system of Linear equations, mentioning the advantages of one method over the other. (6)

(c) Find the inverse of the coefficient matrix of the system

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$$

by Gauss Jordan method with partial pivoting & hence solve the system. (6)

6. (a) Find the unique interpolating polynomial P(x) of degree 2 or less, which interpolates f(x) at the points $x = 0, 1, 4$ such that $f(0) = 1$, $f(1) = 27$, $P(4) = 64$ by

(1). Lagrange's method

(2) Newton's Divided Difference method

Hence evaluate f(3). (6)

(b) Determine an appropriate step size to be used in the tabulation of $f(x) = (1 + x)^6$ in the interval [0, 1], so that the truncation error for linear interpolation is to be bounded by 5×10^{-5}. (6)

(c) Evaluate $\int_0^1 \frac{1}{1+x}\,dx$, correct to three decimal places by Trapezoidal rule and Simpson's $1/3^{rd}$ rule with $h = 0.5$. Also determine which method yields more accurate result & Why ?

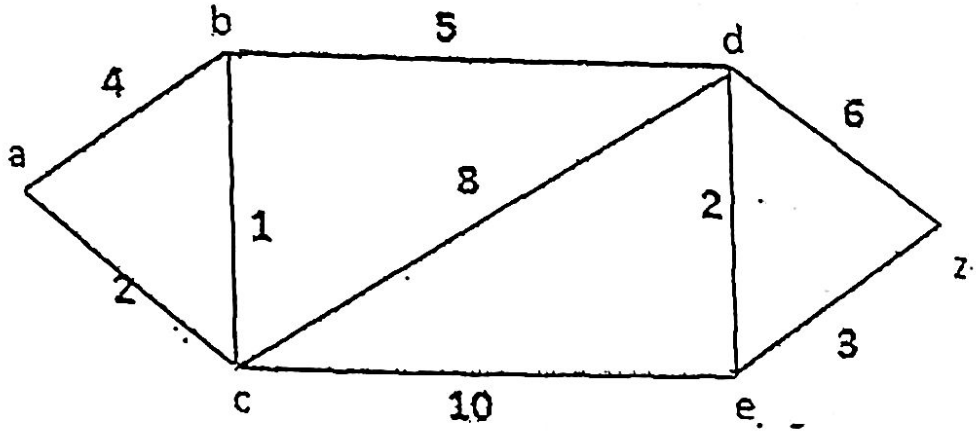
(6)

Unit III(2)

(Discrete Mathematics)

5. (a) Find the length of a shortest path between a and z in the given weighted graph

(6)



(b) Show that if G is a connected planar simple graph, then G has a vertex of degree not exceeding five. Is K_5 planar graph ?

(6)

(c) Define the following terms with example :

(i) A directed graph

P.T.O.

(ii) A Hamilton circuit

(iii) A Eulerian path. (6)

6. (a) Write the following Boolean expression :

$$E(x_1, x_2, x_3) = (x_1 \wedge x_2 \wedge \overline{x_3}) \vee (x_1 \wedge \overline{x_2} \wedge x_4) \wedge (x_2 \wedge \overline{x_3} \wedge \overline{x_4})$$

over the two valued Boolean algebra in conjunctive normal form. (6)

(b) Draw the switching circuit corresponding to the algebraic expression :

$$a \wedge \left[(b \vee \overline{d}) \vee \{\overline{c} \wedge (a \vee d \vee \overline{c})\}\right] \vee b$$ (6)

(c) Show that the following statement is a tautology:

$(A \rightarrow B) \rightarrow [(\overline{A} \rightarrow B) \rightarrow B]$, where $\overline{A}$ denotes the negation of A. (6)

Unit-III(3)

(Mathematical Statistics)

5. (a) Show that for any discrete distribution, standard deviation is not less than mean deviation from mean. (6)

(b) A coin is tossed until a head appears. What is the expectation of the number of tosses required ? (6)

(c) Two independent variables x_1 and x_2 are with means 5 and 10 and variances 4 and 9. If $u = 3x_1 + 4x_2$ and $v = 3x_1 - x_2$, find the correlation coefficient between u and v. (6)

(a) Prove that the recurrence relation for the Poisson distribution with mean λ is $\mu_{r+1} = r\lambda\mu_{r-1} + \lambda\frac{d\mu_r}{d\lambda}$, where μ_r is the r^{th} moment about the mean. Hence deduce the values of β_1 and β_2. (6)

(b) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. Given that if

$f(t) = \frac{1}{\sqrt{2\pi}}\int_0^t \exp\left(-\frac{x^2}{2}\right)dx$, Then $f(0.496) = 0.9$ and $f(1.405) = 0.42$. (6)

(c) For two variables X and Y, the two regression lines are $8X - 10Y + 66 = 0$ and $40X - 18Y = 214$. If $Var(X) = 9$

Calculate (i) the mean values of X and Y

(ii) the correlation coefficient between X and Y.

(6)

Unit III(4)

(Mechanics)

5 (a) Three forces each equal to 'P' act along the sides of triangle ABC in order, prove that the resultant is:

$$P\left[1-8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right]^{1/2}$$

and find the distance of its line of action from one angular point ,and here it cuts one side of the triangle. (6)

(b) Two weights w_1 and w_2 rest on a rough plane inclined at an angle α to the horizontal and are connected by a string which lies along the line of greatest slope. If μ_1 and μ_2 are their coefficients of friction with the plane and $\mu_1 > \tan\alpha > \mu_2$, prove that, if they are both on the point of slipping,

(6)

$$\tan\alpha = \frac{\mu_1 w_1 + \mu_2 w_2}{w_1 + w_2}.$$

3

(c) Find the centre of gravity of a quadrant of a circular disc of radius a. (6)

(a) A particle is performing a S.H.M of period T about a centre 'O' and it passes through a point P, where OP = b with velocity v in the direction OP. Prove that the time which elapses before its return to P is

$$\left(\frac{T}{\pi}\right)\tan^{-1}\left(\frac{vT}{2\pi b}\right). \quad (6)$$

(b) A particle moves under the influence of a centre which attracts with a force :

$$\left(\frac{b}{r^2}+\frac{c}{r^4}\right),$$

'b' and 'c' being positive constants and 'r' the distance from the centre. The particle moves in a circular orbit of radius 'a'. Prove that the motion is stable if and only if, $a^2b > c$. (6)

(c) If v_1 and v_2 be the velocities at the ends of a focal chord of a projectile's path and 'u', the horizontal component of velocity, show that

$$\frac{1}{v_1^2}+\frac{1}{v_2^2}=\frac{1}{u^2} \quad (6)$$

Unit III(5)

(Theory of Games)

5. (a) Solve graphically the LPP :

Minimize $Z = 20x_1 + 10x_2$

subject to :

$x_1 + 2x_2 \leq 40$

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \geq 60$

$x_1, x_2 \geq 0$

(b) Use simplex method to solve following problem :

Maximize $Z = 4x_1 + 10x_2$

subject to : $2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_1 + 3x_2 \leq 90$

$x_1, x_2 \geq 0$

(c) Verify that the dual of dual is primal for the following LPP :

Maximize : $Z = 2x_1 + 5x_2 + 6x_3$

subject to :

$$5x_1 + 6x_2 - x_3 \leq 3$$

$$-2x_1 + x_2 + 4x_3 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

(6)

(a) Use graphical method to solve the rectangular game whose pay off matrix is :

$$\begin{bmatrix} -2 & -6 \\ -4 & -5 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}$$

(6)

(b) Find the range of values of p & q which will render (2,2) a saddle point of the game :

$$\begin{bmatrix} 0 & 2 & 3 \\ 8 & 5 & q \\ 2 & p & 4 \end{bmatrix}$$

(6)

P.T.O.

(c) Reduce the following game to an LPP and hen[ce]
solve :

$$\begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix}$$

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **3051**

Unique Paper Code : **62353424** **GC-4**

Name of the Paper : **Computer Algebra Systems and Related Softwares**

Name of the Course : **B.A. (Prog.) Mathematics Skill Enhancement Course**

Semester : **IV**

Duration : **2 Hours** Maximum Marks : **50**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

1. Fill in the blanks : 10×1

(*i*) The argument of a function in Mathematica is given by ....

(*ii*) The command .... determines the prime factorization in Mathematica.

P.T.O.

30

(*iii*) The command .... is used for matrix exponentiatio[n]

in Maxima.

(*iv*) Thr plotting engine that beneath Maxima is called ......

(*v*) In Maple .... breaks the line.

(*vi*) Lines in Maple may be terminated with .... to

suppress the output of the calculation.

(*vii*) The command .... returns the rank of a matrix in

MATLAB/Octave.

(*viii*) MATLAB stands for ....

(*ix*) The command for $\sqrt{10}$ in R is ....

(*x*) In R, the .... function produces box

plots.

2. Answer any *eight* parts from the following : 8×2½

(*i*) Write the commands in R for the following :

(*a*) Put the list of 2, 5, 6, 10, 11, 9, 1, 0, 6, 8, 7

into a variable B.

(*b*) To sort the array B.

(*c*) To find simple standard deviation of B.

(*ii*) Write the command in R to simulate a random sample of 20 items from a normally distributed data that has mean 50 and standard deviation 8.

(*iii*) Write a program in MATLAB/Octave to plot the graph of a circle $x = \cos t$, $y = \sin t$ for $0 \leq t \leq 2\pi$ with step size 0.01.

(*iv*) Write the output for the following commands :

(*a*) zeros (2, 3)

(*b*) eye (2)

(*c*) ones (3)

(*d*) v = [4; 3; 5]

(*e*) s = [1, 0, 2].

(*v*) Write *two* differences between Maxima and Maple.

(*vi*) Write the use of the following commands in mathematica :

(*a*) PlotLegend

(*b*) Do loop

(*c*) ;

(*vii*) Define and differentiate a function $f(x) = x^2 + \sin x$ in Maxima.

(*viii*) Write a program in Maxima to plot the surface :

$$g(x, y) = \cos x + \sin y$$

for $-1 \le x \le 1$ and $-1 \le y \le 1$.

(*ix*) Write a program to plot the surface $z = e^{-(x^2 + y^2)}$ for $-2 \le x, y \le 2$ in Maple.

(*x*) Write a command to construct a table of the first 50 prime numbers in Mathematica.

3. Answers any *four* parts from the following : 5×4

(*i*) Write a program in R for the following :

(*a*) Put the data into a variable x :

19	8	6	3	9	1
2	8	8	9	2	9
4	6	3	1	8	3
6	3	7	4	7	2
8	4	8	3	2	9

(*b*) Generate a five number summary of x.

(*c*) Create a box plot of x.

(*d*) Create a stem and leaf plot of x

(*e*) Create a normal probability plot of x.

(ii) Write a program in MATLAB/Octave to solve the system of equations :

$$2y - z + 3t = 20$$

$$2x + y + z - t = 29$$

$$-5x + y + 3t = 12$$

$$7x - 2y + 3z + 4t = 18 .$$

(iii) Write the program in Mathematica, to plot the function :

$$f(x) = e^x - \sin x,\ g(x) = x^2 \cos\left(\frac{1}{x^2}\right),$$

$$h(x) = x^3 + x - 1 \text{ for } 0 \le x \le 3.$$

Use the commands for plot legend, color and thickness.

(iv) Explain thru-do Loop in Maxima. Set $c = 1.6 - 0.8i$ and $z = 0$, then write a program in maxima to iterate $f(z) = 2z^2 - c$ ten times.

(v) Write the command in Maple for the following :

(a) Let $A = \begin{pmatrix} -1 & 0 & 3 \\ 2 & 2 & 5 \\ 6 & 3 & 1 \end{pmatrix}$ and

$$B = \begin{pmatrix} 3 & -1 & 1 \\ -2 & 2 & 5 \\ 3 & 6 & 7 \end{pmatrix}$$

(b) Find A + B

(c) Find matrix multiplication of A and B

(d) $\Sigma_{i=1}^{20}(i^2 + i + 1)$

(e) 7^9 mod 8.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1886 GC-4

Unique Paper Code : 42341202

Name of the Paper : Database Management Systems

Name of the Course : **B.Sc. Prog./Mathematical SC**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates:

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **Section A** is compulsory.
3. Attempt any **five** question from **Section B.**

Section A

1. Answer the following: (25)

   (a) What are the responsibilities of DBA (2)

   (b) List two main types of DMLs. What is the main difference between the two? (2)

(c) List the various cases where use of a null value wou[ld] be appropriate. (2

(d) Define the terms: Metadata and Derived attribute. (2

(e) Differentiate between Specialization and Generalizatio[n] (2

(f) What is the distinction between a database schema an[d] a database state? (3

(g) Define normalization. What dependencies are avoide[d] when a relation is in 2NF? (3

(h) List and explain any three unary relational algebra[ic] operations. (3

(i) What is the difference between logical dat[a] independence and physical data independence? (3

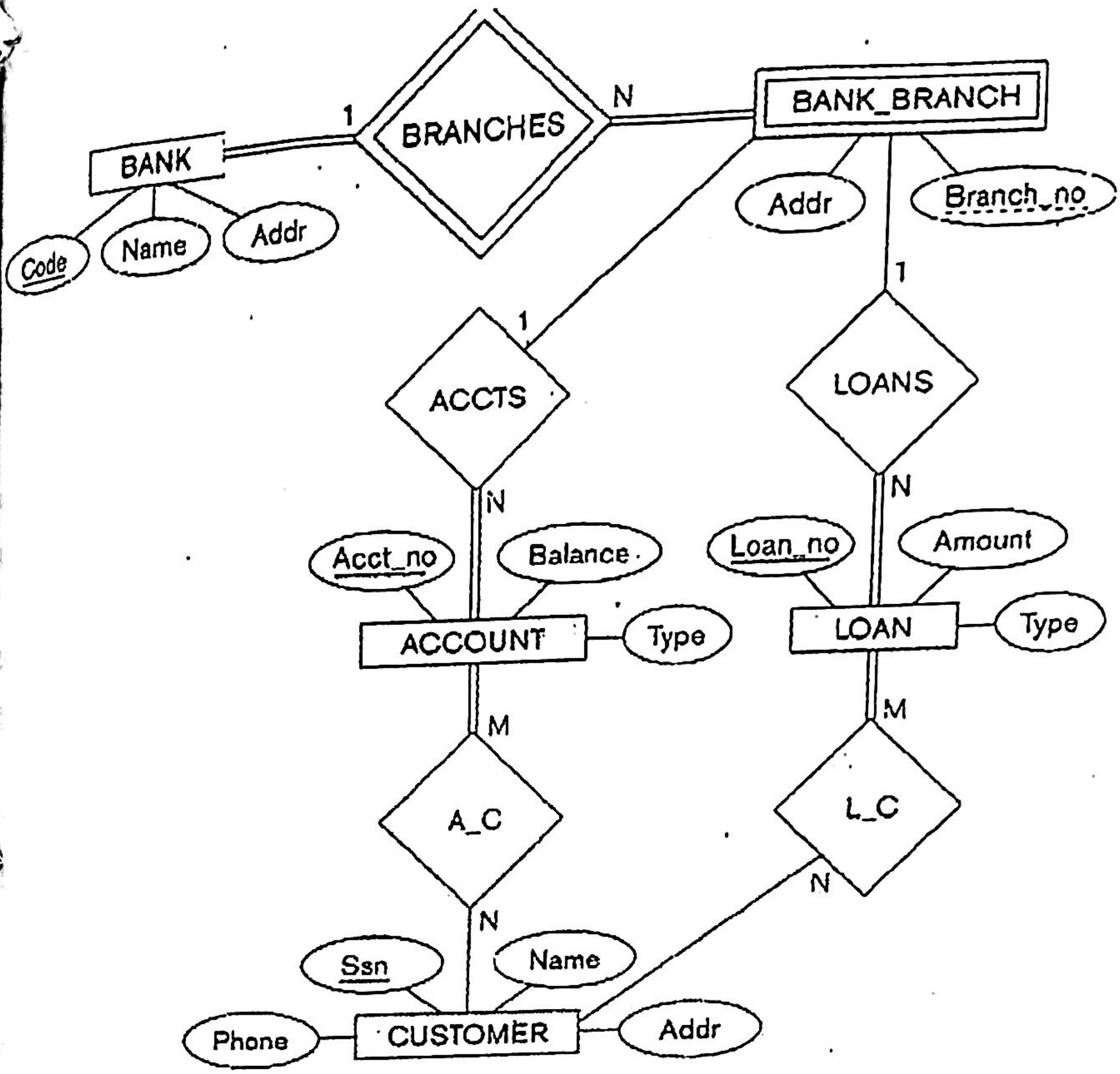
(j) Define foreign key. What is it's purpose? (3

Section B

2. (a) Discuss the main characteristics of the databas[e] approach and how it is different from traditional fil[e] systems. (5

(b) Consider the ER diagram shown below for a BAN[K] database. (5

1886



(i) List the entity types in the ER diagram.

(ii) Give the name of the weak entity type, its partial key, and the identifying relationship.

(iii) Suppose that every customer is restricted to at most two loans at a time. How does this show up on the (min, max) constraints?

P.T.O.

3. (a) Describe the three- schema architecture. Why do w need mappings between schema levels. (5

(b) Consider the following relational database. Give a expression in relational algebra to express each of th following queries(attributes have their usual meaning) (5

EMPLOYEE (**employee_id**, employee_name, street city, salary);

COMPANY (**company_id**, company_name, city);

WORKS (**Emp_id**, Comp_id, hours);

(i) List the Company id and company name of all th companies.

(ii) Find the names of all employees who live in cit **Mumbai** and salary is greater than Rs. 30,000.

(iii) Find the names of all companies based in **Delhi**

(iv) Rename the attributes emp_id and comp_id a employee_id and Company_id respectively.

4. (a) Discuss the entity integrity and referential integrit constraints. Why is each considered important? (5

(b) Consider the universal relation R={A, B, C, D, E, F G, H, I, J} and the following set of functiona dependencies: (5

AB→C

A→DE

B→F

F→GH

D→IJ

What is the key for R? Decompose R into 2NF relations.

. (a) Consider the following relations for a database that keeps track of business trips of salesperson in a sales office (attributes have their usual meanings): (5)

SALESPERSON (Sid, Name, Start_Year, Dept_No)

TRIP (Sid, From_City, To_City, Departure_Date, Return_Date, Trip_Id)

EXPENSE (Trip_Id, Account#, Amount)

Specify the Primary keys and foreign keys for this schema. State any assumptions that you make in identifying the keys.

(b) Explain third normal form. How does it differ from 2NF? (5)

6. (a) Explain with the help of an example how can t following ER model constructs be mapped to t relational database tables: (Any Two) (

(i) Weak Entity Types

(ii) Binary M:N Relationship Types

(iii) Multivalued Attributes

(b) Perform the operations mentioned below on t following tables (attributes have their usual meaning (

Student(<u>rollno</u>, name, age, dno)

Department(<u>dno</u>, dname)

Student

101	**Amit**	**18**	**D1**
102	**Ankit**	**19**	**D3**
103	**Pooja**	**18**	**D1**
104	**Suresh**	**18**	**D2**

Department

D1	**Computer Science**
D2	**Physics**
D3	**Mathematics**

(i) Join

(ii) Cartesian Product

7. Consider the following relational database (attributes hav their usual meaning): (1(

Supplier (S, Sname, Status, City)

Part (P#, Pname, Color, Weight, City)

Project (J#, Jname, City)

Shipment (S#, P#, J#, Qty)

Write SQL commands to express each of the following queries:

(i) Get supplier number and status of all **'Delhi'** suppliers in decreasing order of status.

(ii) Get supplier names for suppliers who supply part **'P2'**.

(iii) For each part supplied, get the part number and total quantity supplied for that part.

(iv) Change the color of part **'P2'** to yellow and increase its weight by 5.

(v) Insert a new tuple into the relation Part.

Explain the following: (10)

(a) Cardinality Ratio

(b) DifferentTypes of Database End Users

(c) Primary key and a super key

(d) Recursive Relationship

(e) Disjointness constraint on specialization

[This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 1894

Unique Paper Code : 42351201 GC-4

Name of the Paper : Calculus and Geometry

Name of the Course : B.Sc. Mathematical Sciences/B.Sc. Physical Sciences

Semester : II

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

Marks of each part are indicated.

1. (*a*) Use $(\in, \delta)$ definition to show that : 6

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6.$$

(*b*) Define uniform continuity. Show that the function f defined by :

$$f(x) = \sqrt{x}, 1 \leq x \leq 3$$

is uniformly continuous. 6

P.T.O.

(c) Let f be a function defined on R by setting :

$$f(x) = |x - 1| + |x + 1|$$

Show that f is not derivable at the points $x = 1$ and $x = -1$, and is derivable at every other point. 6

2. (a) Show that the function f defined on R by setting :

$$f(x) = \begin{cases} \dfrac{e^{1/x}}{1 + e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is discontinuous at $x = 0$. State the kind of discontinuity. 6

(b) State and prove Lagrange's Mean Value Theorem and give its geometrical interpretation. 6

(c) Find the area of the curve $r = 2a \cos \theta$. 6

3. (a) Find the asymptotes of the curve : 6

$$x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0.$$

(b) Determine the position and nature of the double points of the following curve : 6

$$x^3 - 2x^2 - y^2 + x + 4y - 4 = 0.$$

(c) Trace the following curve : 6

$$x = a\cos^3\theta,\ y = a\sin^3\theta.$$

(a) Evaluate the definite integral : 6½

$$\int_1^2 \frac{dx}{x^2\sqrt{x^2-1}}.$$

(b) Trace the curve : 6½

$$x^2(a^2 - x^2) = y^2(a^2 + x^2).$$

(c) If $I_n = \int_0^{\pi/4} \tan^n\theta$, show that : 6½

$$I_n + I_{n-2} = \frac{1}{n-1}$$

Deduce the value of I_5.

(a) Find the volume of the solid obtained by revolving the cardioid : 6

$r = a(1 - \cos\theta)$ about the initial line.

(b) Sketch the parabola $(x+2)^2 = -(y+2)$ and label its focus, vertex and directrix. 6

(c) Find the equation of the hyperbola with vertices $(0, \pm 8)$ and asymptotes $y = \pm\frac{4}{3}x$. 6

6. (*a*) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle of $\theta = 30°$. Find an equation of the curve $\sqrt{3}xy + y^2 = 6$ in $x'y'$-coordinates. 7

(*b*) (*i*) Find the centre and radius of the following sphere : 4

$$x^2 + y^2 + z^2 + 6x - 8y + 10z - 14 = 0.$$

(*ii*) Find :

$$\nabla \times (\nabla \times \vec{F})$$

where :

$$\vec{F} = y^2x\hat{i} - 3yz\hat{j} + xy\hat{k}.$$ 3

(*c*) (*i*) Prove that div(curl F) = 0, where F = F(x, y, z) is a vector field. 4

(*ii*) Prove that curl(grad ϕ) = 0, where $\phi = \phi(x, y, z)$ is a scaler field. 3

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : **2946** **GC-4**

Unique Paper Code : 42354401

Name of the Paper : Real Analysis

Name of the Course : **B.Sc. Mathematical Sciences / B.Sc. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. This question paper has **six** questions in all. Attempt any two parts from each question.

1. (a) State the completeness property of real numbers. Show that the set Q of rational numbers is not complete.

   (b) Let A be a non-empty set of real numbers and is bounded above. If $B = \{x \mid -x \in A\}$, then show that B is bounded below and $\inf B = -\sup A$.

   (c) Define finite, infinite, countable and uncountable sets and give one example of each of these sets. (6,6)

P.T.O.

2\. (a) Define limit point of a set $S \subseteq R$. Show that '0' is the only limit point of the set $S = \left\{\frac{1}{n} : n \in N\right\}$.

(b) Let (x_n) and (y_n) be sequence of real numbers that converge to x and y respectively, such that $y_n \neq 0 \ \forall \ n \in N$ and $y \neq 0$ then show that the sequence $\left(\frac{x_n}{y_n}\right)$ converges to $\frac{x}{y}$.

(c) If $\lim_{n\to\infty} a_n = L$, then prove that $\lim_{n\to\infty} \frac{a_1 + a_2 + \cdots\cdots + a_n}{n} = L$

(6,6)

3\. (a) State Cauchy's Convergence Criterion for sequences and hence check the convergence of (S_n), where

$$S_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} \cdots\cdots + \frac{1}{n!} \ \forall \ n \in N.$$

(b) Let (a_n) be a sequence defined as $a_1 = \frac{3}{2}$, $a_n = 2 - \frac{1}{a_n} \ \forall \ n \geq 1$. Show that (a_n) is monotonically decreasing and bounded below. Hence show that (a_n) converges to 1.

(c) Let $\sum_{n=1}^{\infty} u_n$, be a positive term series such that $\lim_{n\to\infty} u_n^{1/n} = L$ then show that $\sum_{n=1}^{\infty} u_n$ converges if $L < 1$ and diverges if $L > 1$. What happens when $L = 1$? (6½,6½)

4. (a) State and prove the necessary condition for the convergence of an infinite series. Is the condition sufficient ? Justify.

(b) Test the convergence of the following series :

(i) $\sum_{n=1}^{\infty}\left(\sqrt{n^3+1} - n^{3/2}\right)$

(ii) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(c) Define absolute convergence for an infinite series of real numbers. Show that the series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\cdots$ is absolutely convergent for all real x. (6½,6½)

5. (a) State Weierstrass M-test for the convergence of a series of functions and hence test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + n^4x^2}$, for every x.

(b) Show that the sequence (f_n), where $f_n(x) = nxe^{-nx^2}$, $x \geq 0$ is not uniformly convergent in $[0, k]$, $k > 0$.

(c) Determine the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n}$. (6½,6½

6. (a) Prove that a bounded function is integrable on [a, b] i for every $\varepsilon > 0$ there exists a partition P of [a, b] such that $U(P, f) - L(P, f) < \varepsilon$.

(b) Show that the function f defined as

$$f(x) = \frac{1}{2^n}, \text{ when } \frac{1}{2^{n+1}} < x \leq \frac{1}{2^n}, \quad (n = 0, 1, 2, 3, \ldots$$

$$f(0) = 0$$

is integrable on [0, 1].

(c) Show that if a function f defined on [a, b] is continuou then it is integrable on [a, b]. (6,6

This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 54

Unique Paper Code : 234661 G

Name of the Paper : CSPT-606 Database Management Systems

Name of the Course : B.Sc. (Prog.) Physical/Mathematical Sciences

Semester : VI

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory.

Attempt any *four* questions out of the remaining Q. No. 2-Q. No. 7.

Parts of a question must be answered together.

1. (*a*) Explain any *two* major characteristics of the database that distinguish it more appealing than traditional file processing. 3+3=6

(*b*) Briefly discuss the function of a catalogue. 3

(*c*) What are DML and DDL commands ? Give an example for each. 5

(*d*) Define the following terms : 2×3=6

(*i*) Multi-valued Attribute

(*ii*) Recursive Relation.

(*e*) Explain the following keys : 2×2=4

(*i*) Primary key

(*ii*) Foreign key.

(*f*) Consider the following database schema : 3

EMPLOYEE(<u>Emp_id</u>, Dept_no, Ename, Salary, Commission)

DEPARTMENT(<u>DNo</u>, Dname)

Write the SQL query to find the employees who earn the same salary as the minimum salary for departments.

(*g*) What are the advantages Normalization ? 3

(*h*) Consider the relation R = {A, B, C, D, E} with the set of functional dependencies as F = {AB $\rightarrow$ CD, ABC $\rightarrow$ E, C $\rightarrow$ A} and X = {ABC}. Find the closure X^+. Is X a key for relation R. 3+2=5

2. (*a*) What are the responsibilities of Database Designers and Administrators ? 3+3=6

(*b*) Explain the concepts of generalization and specialization in EER diagram. 4

3. (*a*) For the following situation draw an ER diagram : 6

A bank offers loan account, saving account and overdraft account. It operates a number of branches. Clients of the bank can have any number of accounts. More than one client may be able to operate a given account. For every relation specify its attributes, primary keys and foreign keys. Also specify the cardinality ratios.

(*b*) Convert the ER diagram obtained in Q. No. 3 (*a*) to relational database schema. 4

4. (*a*) Consider the following database schema that keeps track of student enrolment in courses and the books in each course : 3×2=6

STUDENT(S#, Name, Major, Bdate)

COURSE(Course#, Cname, Dept)

ENROLL(S#, Course#, Quarter, Grade)

BOOK_ADOPTION(Course#, Quarter, Book_isbn)

TEXT(Book_isbn, Book_title, Publisher, Author)

Specify the primary key and foreign keys for this schema, stating any assumptions you make, where the symbols have usual meaning.

(*b*) Explain Referential Integrity constraint with help of an example. 4

5. Given the following database schemas where the symbols have usual meaning :

EMPLOYEE(<u>SSN</u>, Name, Salary, DeptNo, Gender, SuperSSN)

DEPT(<u>DNo</u>, DName, MrgSSN)

PROJECT(<u>PNo</u>, PName, Plocation)

WORKS_ON(<u>SSN, PNO</u>, hours)

Write Relational Algebra Expressions for the following :

(*a*) Retrieve the average salary of all employees. 3

(*b*) Retrieve the names of all employees who work in 'Production' department. 3

(*c*) Find the names of employees who are supervised by 'Vishal Sharma'. 4

6. Consider the following schema for company database where the symbols have usual meaning :

EMPLOYEE(SSN, Name, Salary, DeptNo, Gender, SuperSSN)

DEPT(DNo, DName, MrgSSN)

PROJECT(PNo, PName, Plocation)

WORKS_ON(SSN, PNO, hours)

Write SQL statements for the following queries :

(*a*) Find the name and SSN of every employee who works for department number 1 and also works on project number 2. 3

(*b*) Find the name and SSN of everyone who works on every project. 3

(*c*) Show the resulting salary if every employee working on 'DBMS' project is given a 10% raise. 4

54

7. Consider a relation given below : 6+2+2=10

R = {I, J, K, L, M, N}

With the following functional dependencies, decompose the given relation into 3NF. Also, find the primary key and prime attributes :

$I \rightarrow J$

$J \rightarrow LN$

$IJ \rightarrow M$

$L \rightarrow N$

1,500

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 43 G

Unique Paper Code : 235266

Name of the Paper : Calculus and Geometry (MAPT-202)

Name of the Course : B.Sc. (Hons.) Computer Science I/B.Sc. Mathematical Sciences/ B.Sc. Physical Sciences I

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates:

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the questions are compulsory.
3. Attempt any **two** parts from each question.
4. Marks of each part are indicated.

1. (a) Use (ε, δ) definition to prove that $\lim_{x\to 3}(x^2-8)=1$. (6)

(b) Show that $f(x) = |x|+|x-1|$ is continuous but not derivable at x= 0 and x = 1. (6)

P.T.O.

(c) State Rolle's theorem and discuss the applicability t $f(x) = x^3 - 4x$ in [-2, 2]. (6)

2. (a) Find the asymptotes of the curve $x(x^2 + y^2) = a(x^2 - y^2)$ (6)

(b) Let f be defined on an interval I. If f be derivable at point $x_0 \in I$, then prove that it is continuous at x_0. (6)

(c) Define uniform continuity of a function and show tha the function defined by $f(x) = x^2, x \in [-1, 1]$ is uniformly continuous. (6)

3. (a) Find the intervals for which the function $y = 2x^4 - 3x$ $+ 2x + 1$ is convex or concave and determine its points of inflexion. (7)

(b) Find the multiple points on the curve $x^4 + y^4 - 2x^2 -$ $2y^2 + 1 = 0$. Also, find the nature of each multiple point (7)

(c) Trace the curve $r = a(1 + \cos\theta)$. (7)

4. (a) Trace the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$. (6)

(b) If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, show that for $n > 1$, $I_n + I_{n-2} =$

$\frac{1}{n-1}$. Deduce the value of I_5. (6)

(c) Draw a rough sketch of the cycloid $x = a(\theta - sin\theta)$, $y = a(1 - cos\theta)$, $0 \leq 0 \leq 2\pi$. Also find the length of the arc of the cycloid. (6)

(a) Find the area of the loop of the curve

$$y^2(a + x) = x^2 (3a - x).$$ (6)

(b) Describe the graph of the equation

$$y^2 - 8x - 6y - 23 = 0.$$ (6)

(c) Find the equation of the hyperbola with vertices $(\pm 2,0)$, foci $(\pm 3,0)$. Describe its reflection property with graph. (6)

(a) Rotate the axes of coordinates to get rid of the xy-term from the equation $x^2 - xy + y^2 - 2 = 0$. Identify the conic and sketch its graph. (6.5)

(b) (i) For $\varphi = \varphi (x, y, z)$, prove that

$$curl(\nabla \varphi) = 0$$ (3)

(ii) For radious vector $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$,, prove that

$$\nabla \frac{1}{\|r\|} = -\frac{r}{\|r\|^3}.$$

(3.5)

(c) (i) Sketch the graph of $z^2 + x^2 = 1$, in 3-space and identify it. (3)

(ii) Using $\frac{d}{dt}[r_1(t).r_2(t)] = r_1(t).r_2'(t) + r_2(t).r_1'(t)$, for

$\overrightarrow{r_1(t)} = 2t\ \hat{\imath} + 3t^2\ \hat{\jmath} + t^3\ \hat{k}$ and $\overrightarrow{r_2(t)} = t^4\ \hat{k}$,

calculate $\frac{d}{dt}\ [r_1(t).r_2(t)]$ (3

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : 276

Unique Paper Code : 235251 G

Name of the Paper : Algebra

Name of the Course : B.A. (Prog.) Discipline Course

Semester : II

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Prove that the set :

$$S = \left\{ \begin{bmatrix} x & y \\ 0 & z \end{bmatrix} : x, y, z \in \mathbf{R} \right\}$$

is a vector space over the field **R** with respect to matrix addition and multiplication of a matrix by a scalar. 6

(*b*) Show that the set :

$$W = \{(a_1, a_2, a_3) : a_1 + a_2 + a_3 = 0, a_1, a_2, a_3 \in \mathbf{R}\}$$

is a subspace of $\mathbf{R}^{(3)}$. 6

P.T.O.

(c) Express the vector $v = (3, 1, -4)$ as a linear combination of :

$a = (1, 1, 1)$, $b = (0, 1, 1)$ and $c = (0, 0, 1)$.

Is $S = \{a, b, c\}$ a basis of $\mathbf{R}^{(3)}$? Justify. 6

2. (a) Solve the system of equations : 6.5

$$x + y + z = 2$$

$$x - 3y + 4z = 3$$

$$3x - 8y + 11z = 11$$

(b) Find the rank of the matrix : 6.5

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 1 & 2 & -3 & 1 \\ -1 & 2 & 3 & -1 \end{bmatrix}.$$

(c) Find the characteristic roots of the matrix : 6.5

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}.$$

3. (a) Show that : 6

$$\sin^5\theta = \frac{1}{16}\{\sin 5\theta - 5\sin 3\theta + 10\sin\theta\}.$$

(b) Sum to n terms : 6

$$\sin\theta + \sin 2\theta + \dots\dots + \sin n\theta.$$

(c) Solve the equation : 6

$$z^{10} - z^5 + 1 = 0.$$

4\. (a) The sum of two of the roots of the equation 6.5

$$x^3 - 5x^2 - 16x + 80 = 0$$

is zero. Find the roots.

(b) Solve the equation 6.5

$$x^3 - 13x^2 + 15x + 189 = 0,$$

being given that one of the roots exceeds another by 2.

(c) If α, β, γ be the roots (such that sum of any of two of them is non-zero) of the equation : 6.5

$$x^3 + qx + r = 0,$$

then find the values of :

(i) $\sum \frac{1}{\beta+\gamma}$

(ii) $\sum \frac{\beta^2+\gamma^2}{\beta+\gamma}$.

5\. (a) Find the multiplicative inverses of the given elements if they exist : [14] in Z_{15} and [35] in Z_{6669}. 6

(*b*) Consider the following permutation in S_7 : 6

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 5 & 6 & 1 & 7 \end{pmatrix} \text{ and}$$

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 6 & 4 & 7 & 3 \end{pmatrix}$$

Compute the following products : $\sigma\tau$ and $\tau^2\sigma$.

(*c*) Prove that any group of prime order is cyclic. 6

6. (*a*) Prove that the rigid motion of an equilateral triangle yields the Group S_3. 6.5

(*b*) If A and B are sub-rings of a ring R, show that $A \cap B$ is also a sub-ring of R. 6.5

(*c*) Let

$$G = \{(a, b) : a, b \in R\}$$

equipped with the binary operation * defined by :

$(a, b) * (c, d) = (a + c, b + d)$ for all $a, b, c, d \in R$,

prove that (G, *) is an abelian Group. 6.5

This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 296

Unique Paper Code : 235451

G

Name of the Paper : Mathematics (Analytical Geometry and Applied Algebra)

Name of the Course : B.A. (Prog.) Discipline Course

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Describe the graph of the equation :

$$y^2 - 8x - 6y - 23 = 0.$$

(*b*) Sketch the ellipse :

$$4x^2 + y^2 + 8x - 10y = -13$$

and label the foci, the vertices, and the ends of the minor-axis.

P.T.O.

(c) Find the centre vertices, foci and asymptotes of the hyperbola whose equation is :

$$4x^2 - 9y^2 + 16x + 54y - 29 = 0$$

and sketch its graph. 6,6,6

2. (a) Find an equation for the parabola whose axis is $y = 0$ and it passes through the points (3, 2) and (2, –3).

(b) Find on equation of the ellipse whose foci are (1, 2) and (1, 4) and minor-axis is of the length 2.

(c) Find an equation for a hyperbola whose foci are (0, ±5) and asymptotes are $y = \pm 2x$. 6,6,6

3. (a) Rotate the coordinate axes to remove the *xy*-term of the curve

$$31x^2 + 10\sqrt{3}\,xy + 21y^2 - 144 = 0$$

and then name the conic.

(b) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle of 45°. Find an equation of the curve

$$3x'^2 + y'^2 = 6$$

in xy-coordinate system.

(c) (i) Find the angle between a diagonal of a cube and one of its edges.

(ii) Find k so that the vector from the point A(1, −1, 3) to the point B(3, 0, 5) is orthogonal to the vector from A to the point P(k, k, k). 6,6,6

4. (a) Find an equation of the sphere that is inscribed in the cube that is centred at the point (−2, 1, 3) and has sides of length 1 that are parallel to the coordinate planes.

(b) (i) Prove that :

$$\left\|\vec{u}+\vec{v}\right\|^2+\left\|\vec{u}-\vec{v}\right\|^2=2\left\|\vec{u}\right\|^2+2\left\|\vec{v}\right\|^2$$

where $\vec{u}$ and $\vec{v}$ are any two vectors.

(ii) Find the vector component of $\vec{a}$ and $\vec{b}$ and the vector component of $\vec{a}$ orthogonal to $\vec{b}$ where

$$\vec{a}=2\hat{i}-\hat{j}+3\hat{k}\cdot$$

$$\vec{b}=\hat{i}+2\hat{j}+2\hat{k}\cdot$$

(c) (i) Find the volume of the tetrahedron with vertices

P(−1, 2, 0), Q(2, 1, −3), R(1, 0, 1), S(3, −2, 3).

(ii) Find two unit vectors that are normal to the plane determined by the points

A(0, −2, 1), B(1, −1, −2) and C(−1, 1, 0) 6,6,6

5. (*a*) Let L_1 and L_2 be the lines whose parametric equations are :

$$L_1 : x = 4t, \quad y = 1 - 2t, \quad z = 2 + 2t$$

$$L_2 : x = 1 + t, \quad y = 1 - t, \quad z = -1 + 4t.$$

Find parametric equations for the line that is perpendicular to L_1 and L_2 and passes through their point of intersection.

(*b*) (*i*) Find the parametric equations of the line that passes through (–1, 2, 4) and is parallel to

$$3\hat{i} - 4\hat{j} + \hat{k}.$$

Also find the intersection of the line with *xy*-plane.

(*ii*) Find an equation of the plane through the point (–1, 4, 2) that contains the line of intersection of the planes :

$$4x - y + z - 2 = 0 \text{ and}$$

$$2x + y - 2z - 3 = 0.$$

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(c) (i) Show that the line

$$x = -1 + t,$$

$$y = 3 + 2t,$$

$$z = -t,$$

and the plane

$$2x - 2y - 2z + 3 = 0$$

are parallel and find the distance between them.

(ii) Find the equation of the plane through the points,

$$P_1(-2, 1, 4),\ P_2(1, 0, 3)$$

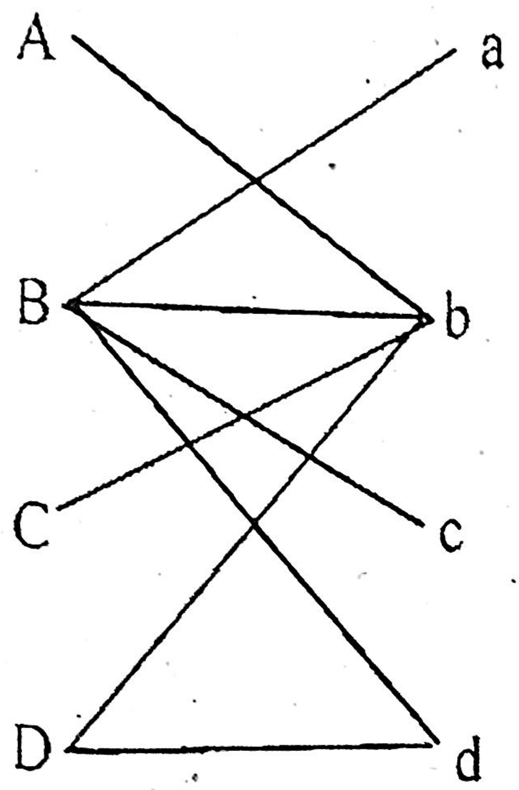
that is perpendicular to the plane :

$$4x - y + 3z = 2.$$

7,7,7

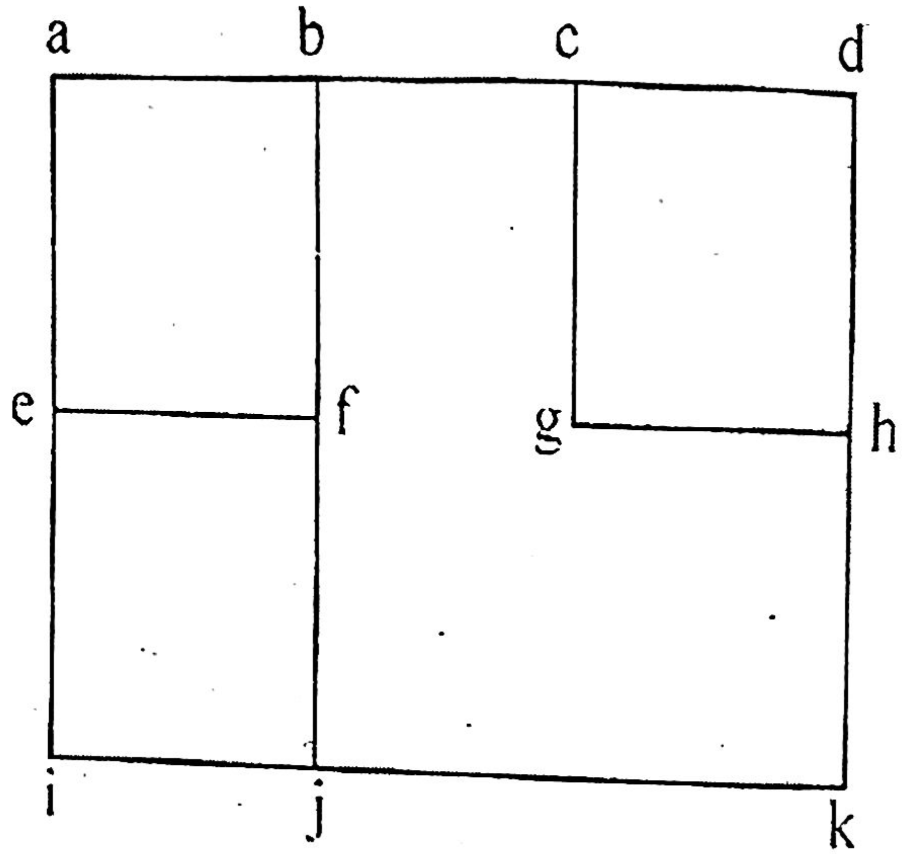
(a) A supermarket wishes to test the effect of putting cereal on four shelves at different heights. Show show to design such an experiment lasting four weeks and using four brands of cereal.

(b) Find a matching or explain why none exists for the following graph :



(c) What are the other sets of 2 edges whose removal disconnects the graph in the following figure besides

(a, b), (a, e) and (c, d), (d, h) ? Either produce other or give an argument why no other exist. 6½,6½,6½



This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **297**

Unique Paper Code : **235451** G

Name of the Paper : **Mathematics (Analytical Geometry and Applied Algebra)**

Name of the Course : **B.A. (Prog.) Discipline Course**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Sketch the parabola :

$$y = 4x^2 + 8x + 5$$

and label the focus, vertex and directrix.

(*b*) Describe the graph of the equation :

$$9x^2 + 4y^2 + 18x - 24y + 9 = 0.$$

P.T.O.

(*c*) Find the centre, vertices, foci and asymptotes of the hyperbola whose equation is :

$$16x^2 - y^2 - 32x - 6y = 57$$

and sketch its graph. 6,6,6

2. (*a*) Find an equation for the parabola that has its vertex at (1, 2) and its focus at (4, 2). Also sketch its rough graph showing the reflection property of parabola at the point (4, –4).

(*b*) Find an equation of the ellipse whose foci are (2, 1) and (2, –3) and the length of its major axis is 6.

(*c*) Find an equation for the hyperbola with vertices (2, 4) and (10, 4) and whose foci are 10 units apart. 6,6,6

3. (*a*) Identify and sketch the curve $xy = 1$.

(*b*) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle of $\theta = 60°$ and then find an equation of the curve :

$$\sqrt{3}xy + y^2 = 6$$

in $x'y'$-coordinates.

(c) (i) Find the component form of $\vec{v}+\vec{w}$ and $\vec{v}-\vec{w}$ in 2-space, given that :

$$\|\vec{v}\| = 1, \|\vec{w}\| = 1, \vec{v}$$

makes an angle of $\frac{\pi}{6}$ with the positive x-axis, and $\vec{w}$ make an angle of $\frac{3\pi}{4}$ with the positive x-axis.

(ii) Find $\vec{u}$ and $\vec{v}$ if

$$\vec{u} + 2\vec{v} = 3\hat{i} - \hat{k} \text{ and}$$

$$3\vec{u} - \vec{v} = \hat{i} + \hat{j} + \hat{k}.$$ 6,6,6

(a) A sphere S has centre in the first octant and is tangent to each of the three coordinate planes. The distance from the origin to the sphere is $3-\sqrt{3}$ units. What is the equation of the sphere ?

(b) (i) Show that two non-zero vectors $\vec{v_1}$ and $\vec{v_2}$ are orthogonal if and only if their direction cosines satisfy :

$$\cos\alpha_1 \cos\alpha_2 + \cos\beta_1 \cos\beta_2 + \cos\gamma_1 \cos\gamma_2 = 0.$$

(ii) Find two unit vectors in 2-space that make an angle of 45° with $4\hat{i} + 3\hat{j}$.

(c) (i) Find the area of the triangle that is determined by the points

$P_1(2, 2, 0)$, $P_2(-1, 0, 2)$ and $P_3(0, 4, 3)$.

(ii) Find two unit vectors that are parallel to yz-plane and are orthogonal to the vector $3\hat{i} - \hat{j} + 2\hat{k}$.

6,6,6

5. (a) Let L be the line whose parametric equations are :

$$L : x = 2t, \quad y = 1 - t, \quad z = 2 + t.$$

Find parametric equations of the line that contains the point P(0, 2, 1) and intersects the line L at a right angle.

(b) (i) Let L_1 and L_2 be two lines whose parametric equations are :

$$L_1 : x = 2 - t, \quad y = 2t, \quad z = 1 + t$$

$$L_2 : x = 1 + 2t, \quad y = 3 - 4t, \quad z = 5 - 2t.$$

Show that L_1 and L_2 are parallel and find the distance between them.

(ii) Find the distance between the given parallel planes :

$$-2x + y + z = 0$$

$$6x - 3y - 3z - 5 = 0$$

(c) (i) Find an equation of the sphere with centre (2, 1, −3) that is tangent to the plane :

$$x - 3y + 2z = 4.$$

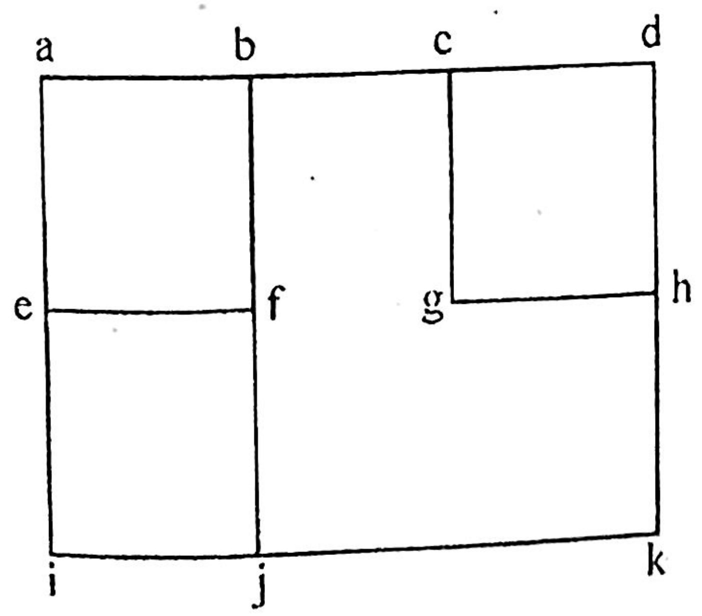
(ii) Find the equation of the plane through (1, 2, −1) that is perpendicular to the line of intersection of the planes :

$$2x + y + z = 2 \quad \text{and}$$

$$x + 2y + z = 3.$$ 7,7,7

6. (a) Construct a Latin square of order 5 on {0, 1, 2, 3, 4}.

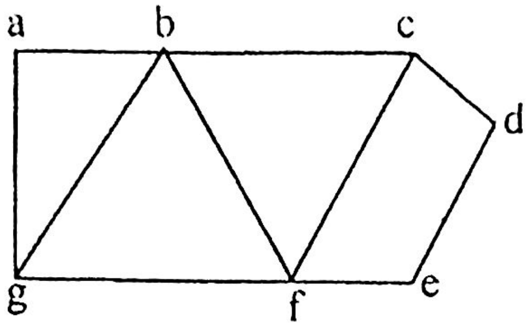
(b) In the following figure find all sets of three corners that have all 11 corners under serveillance. Give a careful logical analysis.



(c) In the following figure find :

(i) All sets of two vertices whose removal disconnects the graph.

(ii) All sets of two edges whose removal disconnects the graph.



6.5,6.5,6.5

This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **347**

Unique Paper Code : **235651** **G**

Name of the Paper : **Numerical Analysis and Statistics**

Name of the Course : **B.A. (Prog.) Discipline Course**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has *six* questions in all.

Attempt any *two* parts from each question.

All questions are compulsory.

Use of scientific calculator is allowed.

Candidate can ask for log/statistical table.

1. (*a*) (*i*) Perform three iterations by Bisection method to obtain the smallest positive root of the equation :

$$f(x) = x^3 - x - 4 = 0.$$

(ii) If a root of $f(x) = 0$ lies in the interval (a, b), then what is minimum number of iterations required when the permissible error is ε. 6

(b) A real root of the equation :

$$f(x) = x^3 - 5x + 1 = 0.$$

lies in the interval (0, 1). Perform four iterations of Secant method to obtain this root. 6

(c) Perform five iterations by Newton-Raphson method to find the root of $N^{1/2}$, where N = 17. Take initial approximation $x_0 = 3$. 6

2. (a) Consider the system of equations :

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

where a is a real constant. For which values of a the Gauss-Seidel method converges. 6

(b) Solve the following system of equations :

$$2x_1 + x_2 + x_3 - 2x_4 = -10$$

$$4x_1 + 0x_2 + 2x_3 + x_4 = 8$$

$$3x_1 + 2x_2 + 2x_3 + 0x_4 = 7$$

$$x_1 + 3x_2 + 2x_3 - x_4 = -5$$

Using the Gauss-elimination method with partial-pivoting. 6

(c) For the following system of equations :

$$-3x_1 + x_2 + 0x_3 = -2$$

$$2x_1 - 3x_2 + x_3 = 0$$

$$0x_1 + 2x_2 - 3x_3 = -1$$

(i) Show that Jacobi iteration scheme converges.

(ii) Starting with $X^0 = [0, 0, 0]^T$, iterate three times. 6

3\. (*a*) Find the unique polynomial of degree 2 or less such that :

$$f(1) = 1, f(3) = 27, f(4) = 64,$$

using Lagrange interpolating formula. Estimate $f(2)$. 6½

(*b*) For the following data :

$$f(0) = 1, f(1) = 14, f(2) = 15,$$

$$f(4) = 5, f(5) = 6, f(6) = 19$$

Obtain the polynomial using Newton divided difference interpolation. Estimate $f(3)$. 6½

(*c*) If

$$f(x) = 1/x,$$

find the divided difference $f[x_1, x_2, x_3, x_4]$. 6½

4\. (*a*) Calculate the coefficient of correlation from the following observations : 6

X	Y
2.52	550
2.49	610

2.47	730
2.42	870
1.69	880
3.43	930
4.72	400

(*b*) Determine the line of regression of Y on X for the following data : 6

X	**Y**
65	67
66	68
67	65
67	68
68	72
69	72
70	69
72	71

(c) Let the pmf $p(x)$ be positive at $x = -1, 0, 1$ and zero elsewhere. If

$$p(0) = \frac{1}{4} \text{ and if } E(X) = \frac{1}{4},$$

determine $p(-1)$ and $p(1)$. 6

5. (a) Let X have the pdf

$$f(x) = \frac{1}{x^2}, \ 1 < x < \infty,$$

zero elsewhere. Show that E(X) does not exist. 6½

(b) Let X be a random variable such that

$$E[(X - b)^2]$$

exists for all real b. Show that

$$E[(X - b)^2]$$

is minimum when $b = E(X)$. 6½

(c) Determine the mode of normal distribution. 6½

6. (a) In a book of 520 pages, 390 typo-graphical errors occur. Assuming Poisson law for the number of error per page,

find the probability that a random sample of pages will contain no error.

(Given, $e^{-0.75} = 0.473$). 6½

(*b*) If X is a normal variate with mean 30 and S.D. 5. Find the probabilities that : 6½

(*i*) $26 \leq X \leq 40$

(*ii*) $X \geq 45$. 6½

(*c*) Determine the moment generating function of Binomial distribution. 6½